Can linear transformations bend a straight line? Comment on Author Correction to Magnetic field screening in hydrogen-rich high-temperature superconductors

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Ref. [1] claims to show magnetization measurements that demonstrate that sulfur hydride and lanthanum hydride under pressure are high temperature superconductors. In Ref. [2], it was pointed out that Figs. 3a and 3e of Ref. [1] were inconsistent with each other according to the figure caption and text in Ref. [3]. Recently, the authors of Ref. [1] published an “Author Correction” [4] to the original form of Ref. [1] explaining that several linear transformations were used to obtain the data published in Fig. 3a from measured data published in Fig. 3e. Here I show that data shown in Fig. 3a of Ref. [1] are not related by any set of linear transformations to data published in Fig. 3e of Ref. [1], contrary to what is stated in the “Author Correction” [4] and in the corrected Ref. [1]. Implications of this finding are discussed.

I. INTRODUCTION

A linear transformation that leads from a set of numbers \( \{x_i\} \) to another set of numbers \( \{y_i\} \) is of the form

\[
y_i = m_1 x_i + b_1.
\]

If we perform another linear transformation on the set of numbers \( \{y_i\} \) to yield a set of numbers \( \{z_i\} \)

\[
z_i = m_2 y_i + b_2
\]

the transformation that relates \( \{z_i\} \) to \( \{x_i\} \) is

\[
z_i = m_1 m_2 x_i + m_2 b_1 + b_2 = m_3 x_i + b_3
\]

and is also a linear transformation.

If the set of points \( \{x_i\} \) are results of measurements performed as a function of magnetic field with values \( H_i \) and they are linearly related to \( \{H_i\} \), they will lie on a straight line when plotted as function of \( H_i \) and obey the relation

\[
x_i = m_1 H_i + b_1
\]

and if we perform any number of linear transformations on the measured data \( \{x_i\} \) to obtain processed data \( \{z_i\} \), the processed data will also be related to the magnetic field by a linear relation

\[
z_i = m_2 H_i + b_2
\]

and lie on a straight line when plotted versus \( H_i \).

The left panel of Fig. 1 shows results for magnetic moment versus magnetic field for a sample of \( H_3S \) under high pressure claimed to become superconducting at temperature \( T_c = 196K \), published as Fig. 3a in Ref. [1]. It was argued in Ref. [1] that the behavior shown in that figure shows that the sample is superconducting. Values for the lower critical magnetic field as function of temperature were obtained from the values of magnetic field where the curves on the left panel of Fig. 1 start to deviate from linearity, those values were shown as Fig. 3c in Ref. [1].

The right panel of Fig. 1 shows portions of hysteresis loops for the magnetic moment at three temperatures, published as Fig. 3e in Ref. [1]. Here we focus on the data for 100K, shown in blue. The middle curve is the virgin curve, obtained when the field is first turned on (the curves were shifted vertically for display), and the upper and lower blue curves are the hysteresis curves obtained when the field is lowered back to zero and negative values, and then increased again to positive values respectively.

The figure caption of Fig. 3 and related text in Ref. [1] in the original form of Ref. 1 [3], published June 9, 2022, implied that the data in Fig. 3a were measured data. We pointed out in Refs. [2, 5] that the data shown in Fig. 3a and in the virgin curves of the Fig. 3e hysteresis loops should be identical. On September 1, 2023, a correction to Ref. [1] was published, Ref. [4], explaining that the original version [3] was in error and that the data shown in Fig. 3a were not the same as those in Fig. 3e because the former were obtained by performing a set of linear transformations on the measured data of...
II. ANALYSIS

A portion of the data shown on the right panel of Fig. 1, for the lowest two blue curves, is shown in Fig. 2. We have added a straight red line connecting the points for magnetic field values 30 mT and 100 mT of the middle curve which is the virgin curve. It can be seen that all the points lie approximately on this straight line, with no systematic deviation from it.

In Fig. 3 we show a portion of the data shown on the left panel of Fig. 1, containing the 100K points (in light blue) for magnetic field between 30 mT and 100 mT. We have drawn a straight red line connecting the light blue points for 30 mT and 100 mT. It can be seen that there is a systematic deviation of the light blue points from this red line. In other words, the data are not linearly related to the magnetic field in this range of fields.

The vertical scales in Fig. 2 and Fig. 3 are rather different, so to make a more direct comparison we plot in Fig. 4 the data of Fig. 2 and Fig. 3 with the same horizontal and vertical scales. The former are clearly approximately linear in field, the latter are clearly not. This implies that the latter have not been obtained from the former by a set of linear transformations.

III. IMPLICATIONS

The “Author Correction” [4] states, in explaining the relation of the data in Fig. 3e to the data in Fig. 3a of Ref. [1], what is now the figure caption of Fig. 3 of Ref. [1]:

Fig. 3e. Ref. [1] was replaced with its corrected version on that date, which includes some information on these transformations. In the original form of the paper (Ref. [3]) there was no information that such transformations had been performed.

We have recently discussed [6] the implications of the Author Correction [4] to the claims of Ref. [1] that $H_{3S}$ is a superconductor and argued that the inferred behavior of magnetic moment with magnetic field is in fact inconsistent with the conclusion that the system is a superconductor.

In this Comment we focus on the relation between measured data and derived data, i.e. Fig. 3e and Fig. 3a of Ref. [1], right and left panels of Fig. 1, for temperature $T=100$K. In Fig. 1 we have drawn a two-sided arrow showing the data that we will be discussing. We will show that the data shown in Fig. 3a are not related to the data shown in Fig. 3e by linear transformations, contrary to what the authors state in Refs. [4] and [1]. The implications of this are then discussed.
Importantly, these linear manipulations do not affect the onset of the deviation of the $M(H)$ virgin curve from the linear dependence.”

Note that the subtraction of a linear background is also a linear transformation. It is clear that the determination of $H_p$ hinges on the crucial fact that all these transformations and “linear manipulations” performed were linear, since $H_p$ was determined by the deviation of the transformed data from linearity. If any of the transformations performed in going from the measured data to the data from which $H_p$ was inferred was non-linear, the procedure would clearly be invalid.

We have shown here that not all transformations performed could have been linear, since from the linear data in Fig. 2, non-linear data in Fig. 3 were derived by these transformations. This renders the determination of $H_p$, a key conclusion of Ref. [1], invalid.

IV. OTHER ISSUES

In order to understand the relation between the data published in Fig. 3a of Ref. [1] and the measured data for all temperatures reported we requested the measured data repeatedly from the authors and from the journal on January 11, 2023 and thereafter. We have not received them. The journal editor, Dr. Prabhjot Saini, informed us that the authors had shared the data with the editors “in confidence” and had stated that our request for data was an “unreasonable request”, hence not covered by the data availability statement that the paper contains. Furthermore, Dr. Saini informed us that the journal could not publish an Editor Note informing readers that there are restrictions on data availability because the editors have received the data.

We have informed the director of the Institute where the measured data for the research reported in Ref. [1] were obtained (Max Planck Institute for Chemistry, Mainz, Germany), Prof. Jos Lelieveld, on April 2, 2023, that the measured data are not being shared. On August 28 2023 Prof. Lelieveld informed us that their institute fully supports the work of Mikhail Eremets and his team.

V. CONCLUSIONS

In order to have confidence that Figs. 3a and 3b of Ref. [1], claimed to be evidence that $H_3S$ and $LaH_{10}$ under pressure are high temperature superconductors, reflect the real physics of the samples under study, it is important to understand the relation between the data shown in those figures and the measured data. We have shown here that for temperature $T = 100K$ the data in Fig. 3a were not obtained by linear transformations from measured data as claimed by the authors. We have also shown in Ref. [6] that the data for $T = 160K$ in Fig. 3a were not obtained from the raw data for that temperature shown in Fig. S12c of Ref. [1] as was claimed in
the paper. For other temperature curves in Figs. 3a and 3b it can also be seen that the relation with the data in Figs. 3e, 3f or S10, S11 is non-linear.

If the data shown in Fig. 3a and Fig. 3b of Ref. [1] do not reflect the real physics of the samples under study, the claim of Ref. [1] that they demonstrate that the systems are high temperature superconductors falls apart. Ref. [1] was the strongest magnetic evidence presented to date that hydrides under pressure are high temperature superconductors [7].

To validate the authors’ interpretation of the implications of the results presented in Figs. 3a and b of Ref. [1] it is imperative that they release the underlying measured data and a detailed explanation of how the published data derived from the measured data. In the absence of this, we argue that Ref. [1] has no credibility and should be retracted from the scientific literature.

Given these facts, the fact that the authors of Ref. [1] decline the request to make their underlying data available to readers and characterize such request as ”unreasonable” indicates that they are unwilling to disclose the details of the process that they used to go from measured data to published data. The reader will form their own conclusions as to why the authors are unprepared to share their data.

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[3] The original version of Nat Commun 13, 3194 (2022), which was on-line between 06/09/2022 and 08/31/2023, is no longer available at the journal’s website but can be found at https://jorge.physics.ucsd.edu/e2021p.pdf.
[5] Ten emails were sent from this author to authors of Ref. [1] asking for clarification of the relation between Figs. 3a and 3e of Ref. [3]. First email October 18, 2022, 10th email May 10, 2023. No clarification was provided.