Dynamics of the Meissner Effect: How Superconductors Expel Magnetic Fields

J. E. Hirsch

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The Meissner effect presents us with a fundamental puzzle that has surprisingly not been noticed before: *how is the mechanical momentum of the supercurrent that expels the magnetic field compensated, so that momentum conservation is not violated?*

The only possible answer is, the body as a whole has to acquire equal and opposite momentum to the one developed by the supercurrent. In a cylindrical geometry, the supercurrent has mechanical angular momentum *parallel* to the applied magnetic field, hence the body has to acquire angular momentum *antiparallel* to the applied field. How does that happen?

The Faraday electric field that develops in the process of magnetic field expulsion transmits angular momentum to the body in the wrong direction, *parallel* to the magnetic field, of magnitude that is many orders of magnitude too large. How does the body manage to ignore this enormous Faraday torque and rotate in the opposite direction?

Any momentum transfer between electrons and the body as a whole has to occur without entropy generation since the transition is thermodynamically reversible. This excludes scattering processes involving impurities or phonons, that generate entropy.

The theory of hole superconductivity [1] explains this puzzle [2]. The explanation relies on the facts that within this theory (a) normal metals becoming superconducting expel electrons from the interior to the surface [3], and (b) the normal state charge carriers are necessarily holes [4].

The conventional theory of superconductivity does not have those physical ingredients, hence we argue that it cannot explain this puzzle. Therefore we argue that superconducting materials described by the conventional theory of superconductivity would either (i) not expel magnetic fields or (ii) violate momentum conservation. Consequently, they don't exist.

The alternative theory of hole superconductivity explains superconductivity as arising through pairing of hole carriers driven by lowering of kinetic energy [5], predicts that superconductors have inhomogeneous macroscopic charge distribution with more negative charge near the surface and more positive charge in the interior [3], and that a spin current flows near the surface in the absence of applied fields [6]. It also provides guidelines for the search for new and better superconducting materials.

[1] References in https://jorge.physics.ucsd.edu/hole.html.

[2] J. E. Hirsch, Europhys. Lett. **114**, 57001 (2016); Annals of Physics **373**, 230 (2016); Phys. Rev. **B 95**, 014503 (2017 IJMPB **32**, 1850158 (2018).

[3] J. E. Hirsch, Phys. Rev. **B 68**, 184502 (2003).

[4] J. E. Hirsch, Phys. Lett. **A134**, 451 (1989).

[5] J. E. Hirsch and F. Marsiglio, Phys. Rev. **B 39**, 11515 (1989); **B 62**, 15131 (2000).

[6] J. E. Hirsch, Ann. Phys. (Berlin) **17**, 380 (2008).

Dynamics of the Meissner effect: how superconductors expel magnetic fields

J.E. Hirsch, UCSD M2S-2018 Beijing

How does the supercurrent that expels the magnetic field start and stop, without violating momentum conservation?

YBaCuO, LiFeAs, FeSe, MgB_2, UPt_3, Sr_2RuO_4, twisted graphene, V_3Si, K_3C_{60}, Pb, Al, Nb, all exhibit a Meissner effect

By understanding the Meissner effect, we will learn something that is relevant to ALL superconductors

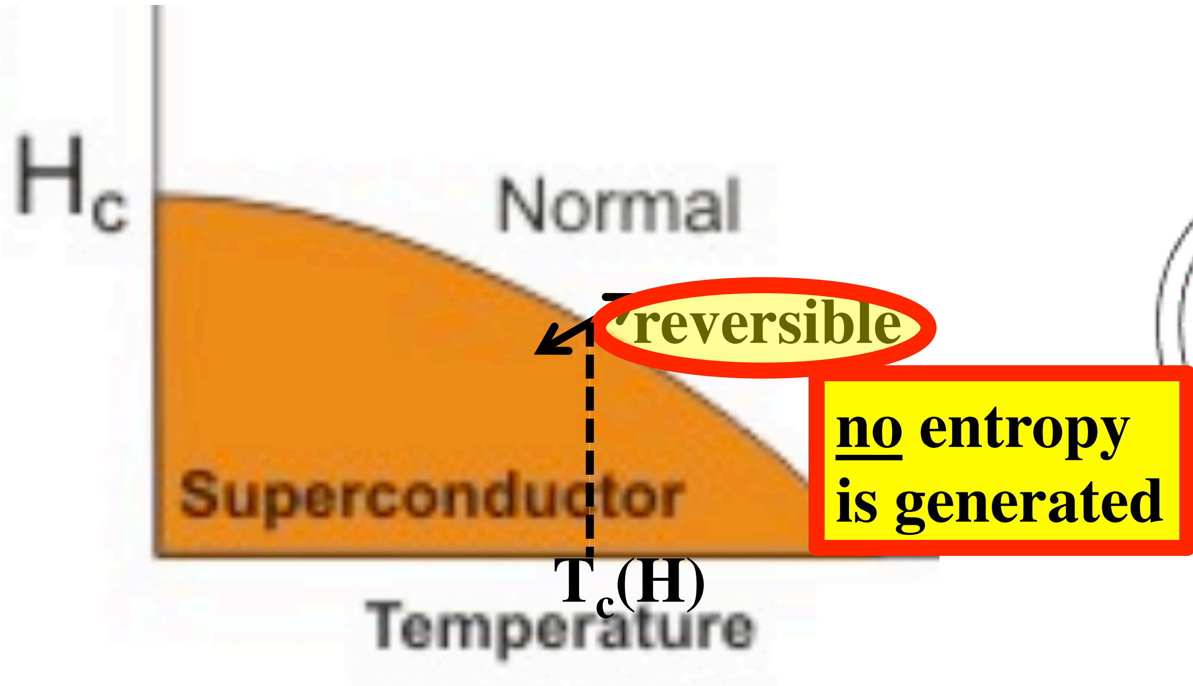
Dynamics of the Meissner effect: how superconductors expel magnetic fields

J.E. Hirsch, UCSD M2S-2018 Beijing

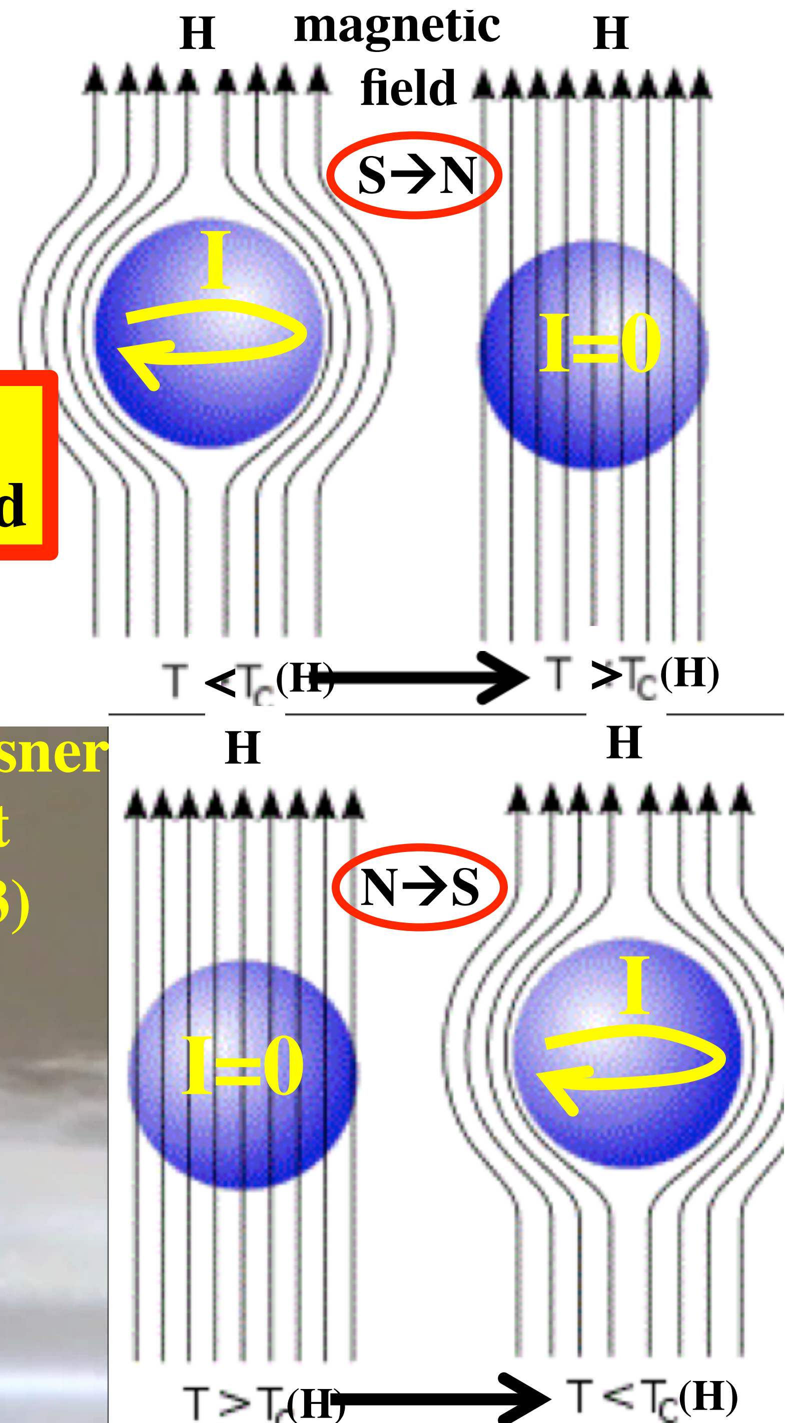
Only materials that have hole carriers can expel magnetic fields

YBaCuO, LiFeAs, FeSe, MgB_2, UPt_3, Sr_2RuO_4, twisted graphene
V_3Si, K_3C_{60}, Pb, Al, Nb, all exhibit a Meissner effect

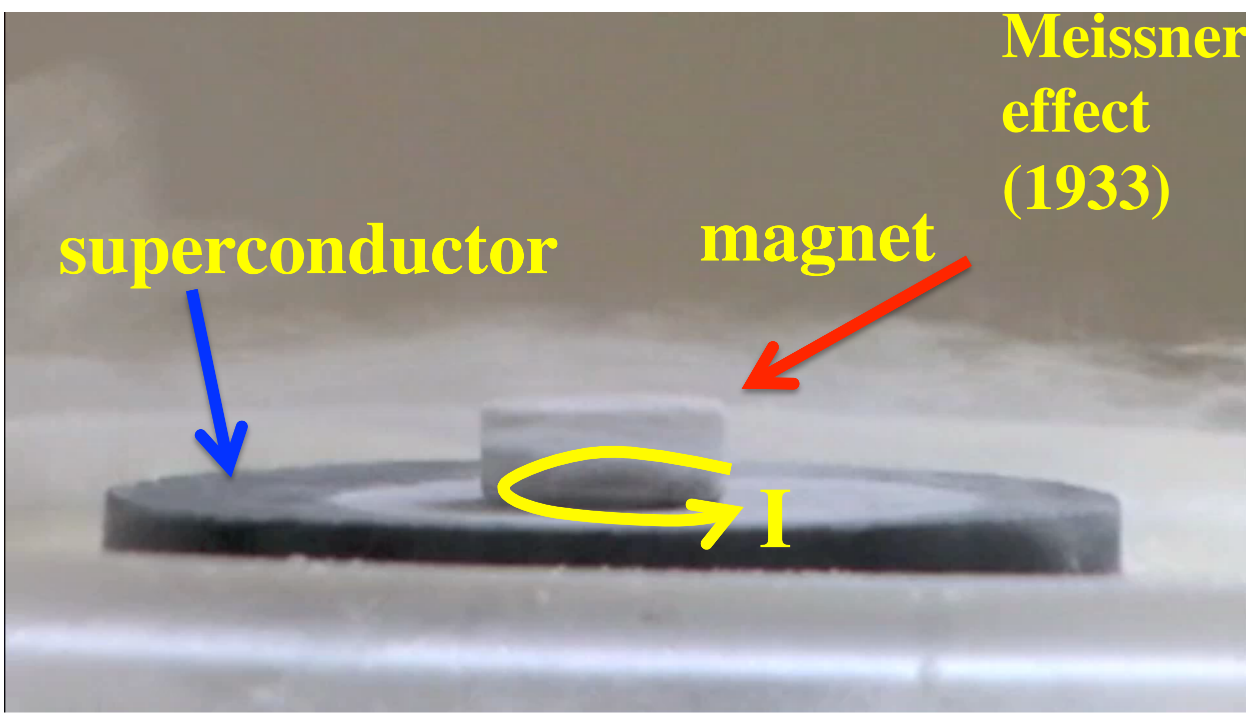
By understanding the Meissner effect, we will learn something that is relevant to ALL superconductors



Hc
Normal
reversible
no entropy is generated
Superconductor
Tc(H)
Temperature



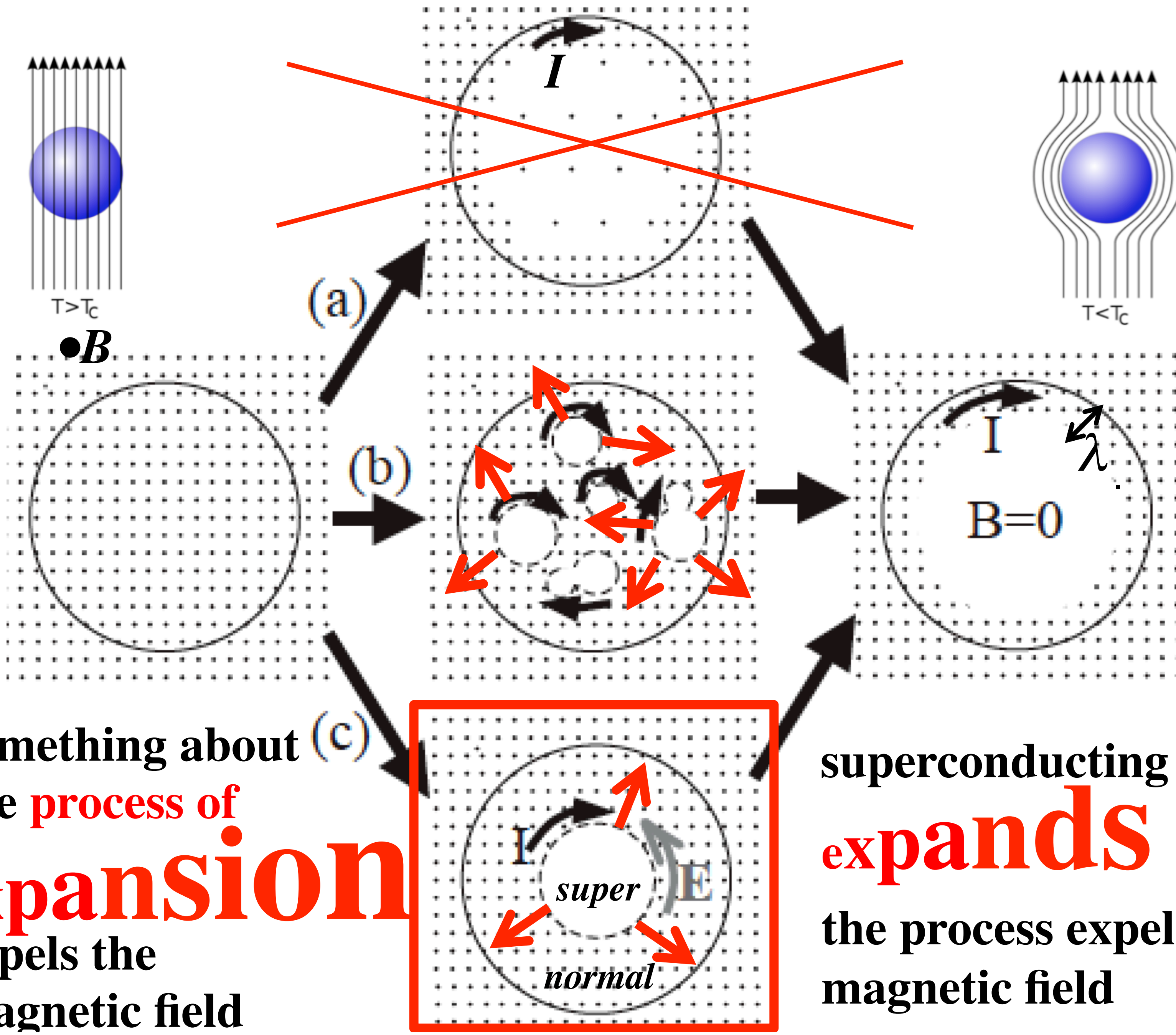
H
magnetic field
H
S→N
I
I=0
T < Tc(H)
T > Tc(H)
H
H
N→S
I=0
I
T > Tc(H)
T < Tc(H)



Meissner effect (1933)
superconductor
magnet
I

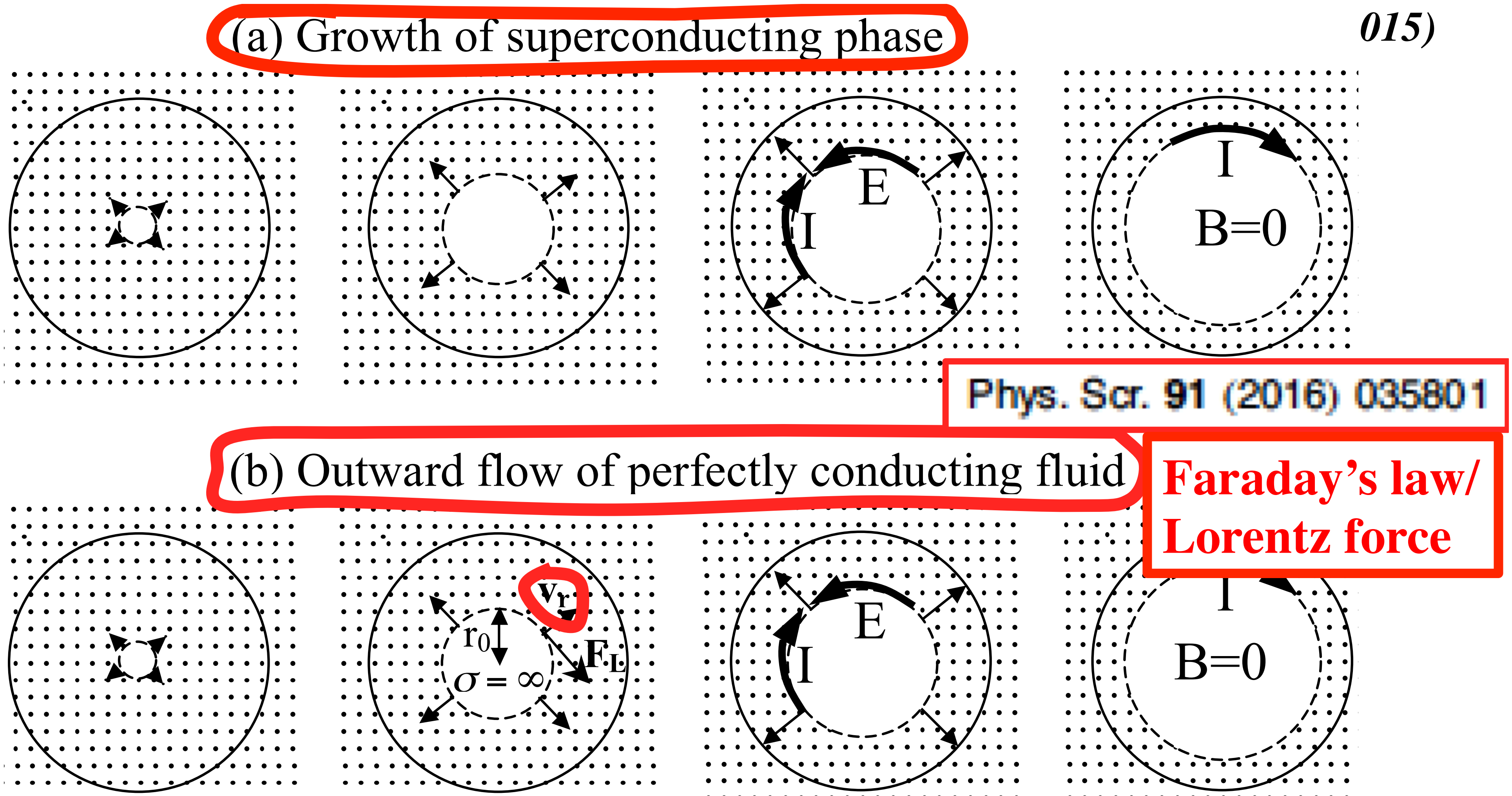
Meissner effect kinetics

JEH, Annals of Physics 362, 1 (2015)



something about the process of expansion expels the magnetic field

superconducting region expands and in the process expels the magnetic field



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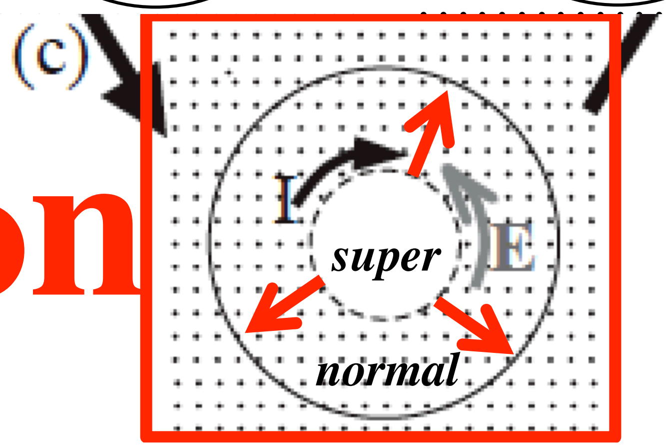


superconducting region expands and in the process expels the magnetic field

What does BCS explain?

* That in the superconducting state, there cannot be a magnetic field in the interior (phase coherence)

* That the energy is lower in the superconducting state with no B than in the normal state with B

$$\psi_{BCS} = |\psi| e^{i\theta(r)}$$

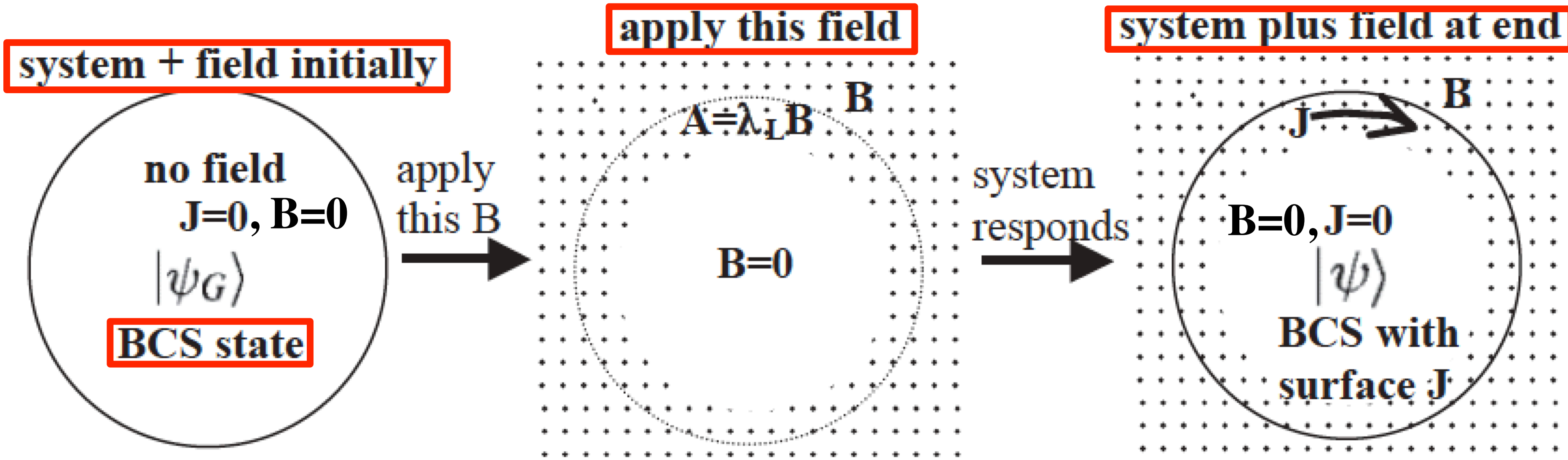
$$\frac{\hbar}{i}\vec{\nabla} = \vec{p} = m\vec{v}_s + \frac{e}{c}\vec{A}$$

$$\vec{p} = 0$$

$$\Longrightarrow \vec{v}_s = -\frac{e}{mc}\vec{A} \Longrightarrow \vec{J} = -\frac{c}{4\pi\lambda_L}\vec{A} \Longrightarrow \vec{\nabla}\times\vec{J} = -\frac{c}{4\pi\lambda_L}\vec{B} \Longrightarrow \nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}$$

* BCS says nothing about HOW the magnetic field gets expelled to reach the final state with $\vec{p} = 0$.

WHAT BCS EXPLAINS



a magnetic field, as shown in Fig. 1 below. The perturbing Hamiltonian is the linear term in the magnetic vector potential $\vec{A}$ that results from the kinetic energy $(\vec{p}-(e/c)\vec{A})^2/2m$, and has the form

$$H_1 = \frac{ie\hbar}{2mc}\sum_i(\vec{\nabla}_i \cdot A + \vec{A}\cdot\vec{\nabla}) \tag{1}$$

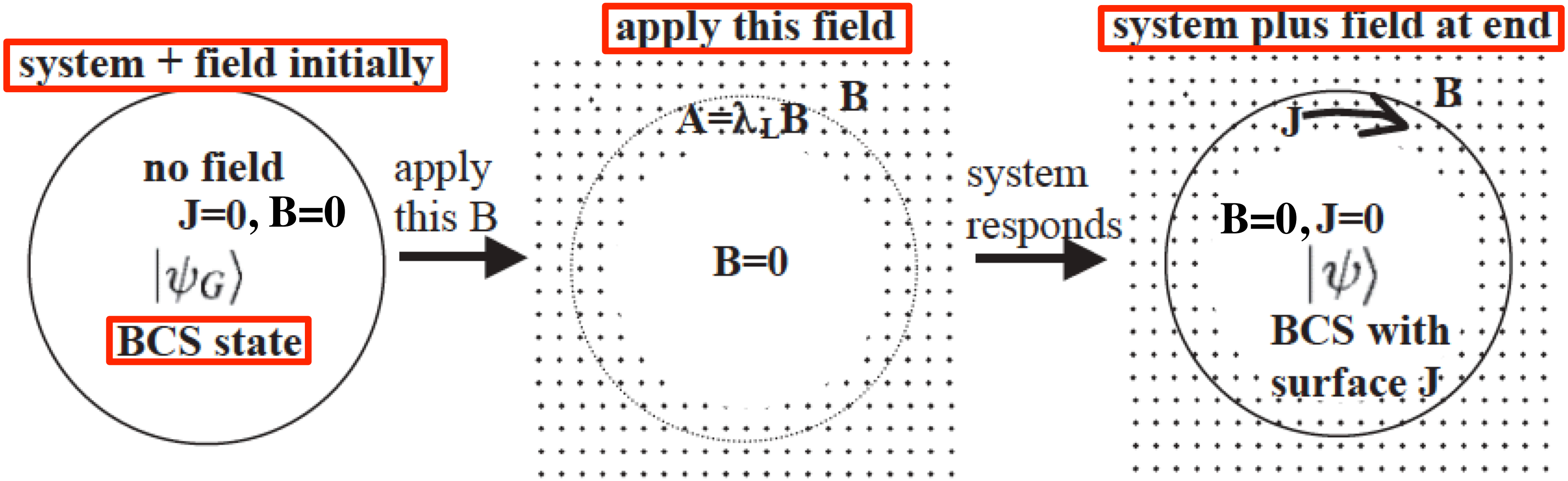
This perturbation causes the BCS wavefunction $|\Psi_G>$ to become, to first order in $\vec{A}$

$$|\Psi> = |\Psi_G> - \sum_n \frac{<\Psi_n|H_1|\Psi_G>}{E_n}|\Psi_n> \tag{2}$$

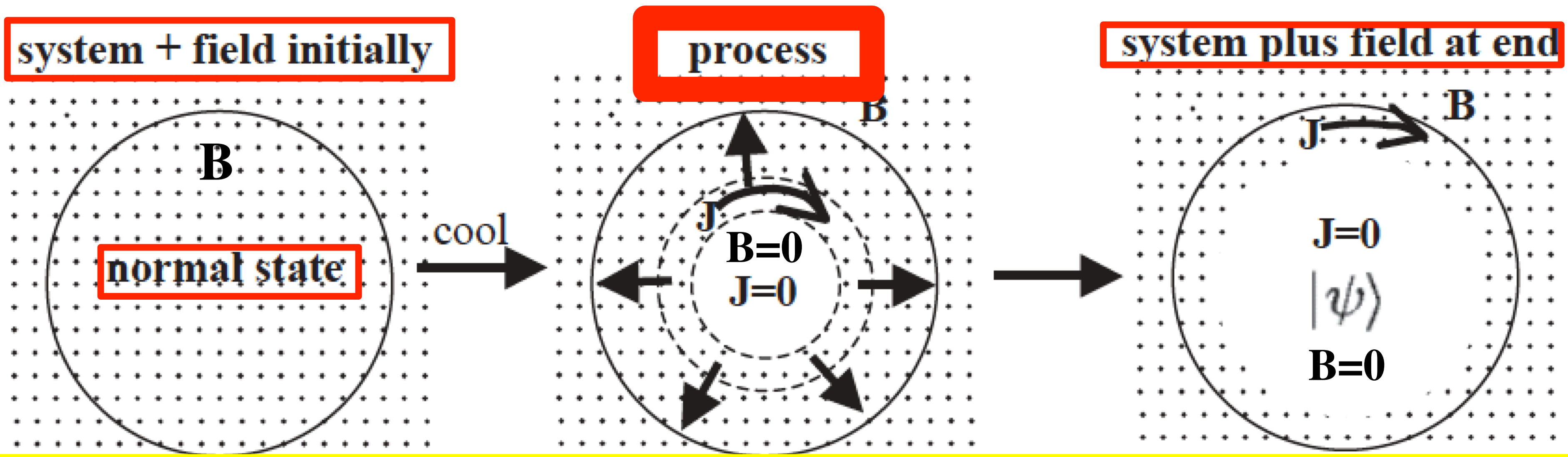
where $|\Psi_n>$ are states obtained from the BCS state $|\Psi_G>$ by exciting 2 quasiparticles, and E_n is the excitation energy. The expectation value of the current operator with this wave function gives the electric current $\vec{J}$:

$$<\Psi|\vec{J}|\Psi> = -\frac{c}{4\pi}K\vec{A} \qquad K = \frac{1}{\lambda_L^2} \tag{3}$$

WHAT BCS EXPLAINS



WHAT THE MEISSNER EFFECT IS

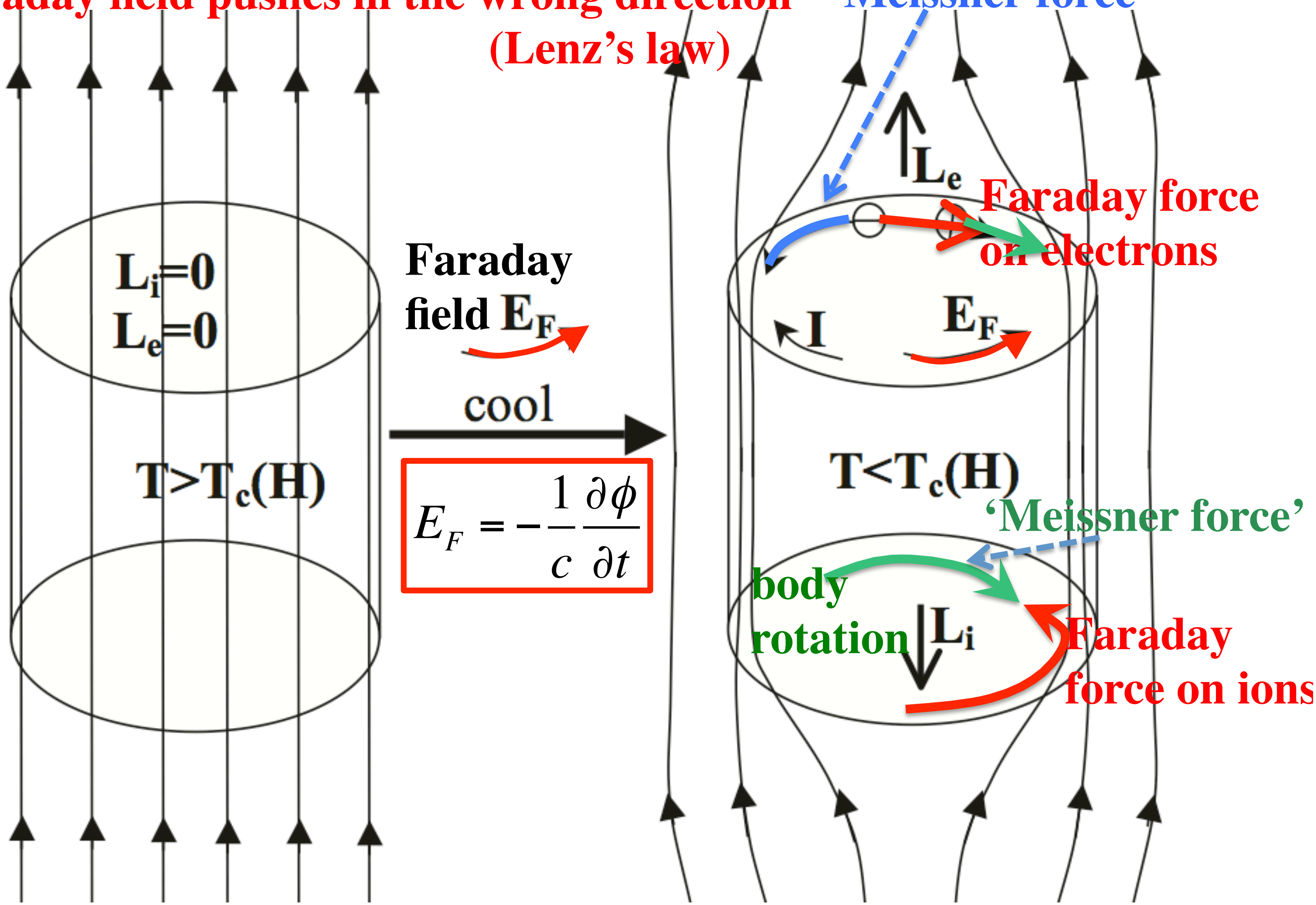


BCS has not AND cannot explain the process

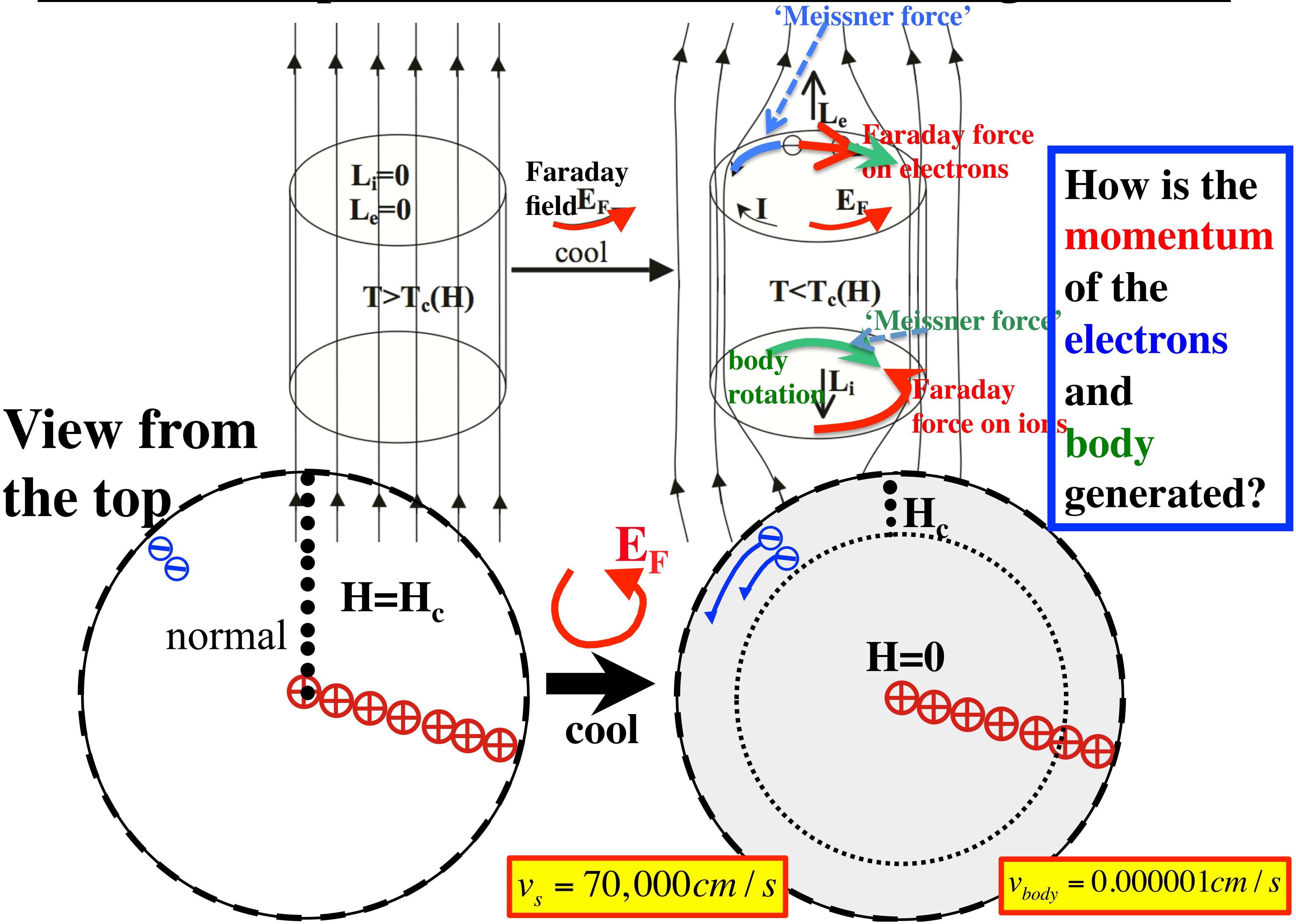
Normal to superconductor transition in a magnetic field

How is the momentum of the electrons (L_e) and body (L_i) generated?

Faraday field pushes in the wrong direction (Lenz's law)

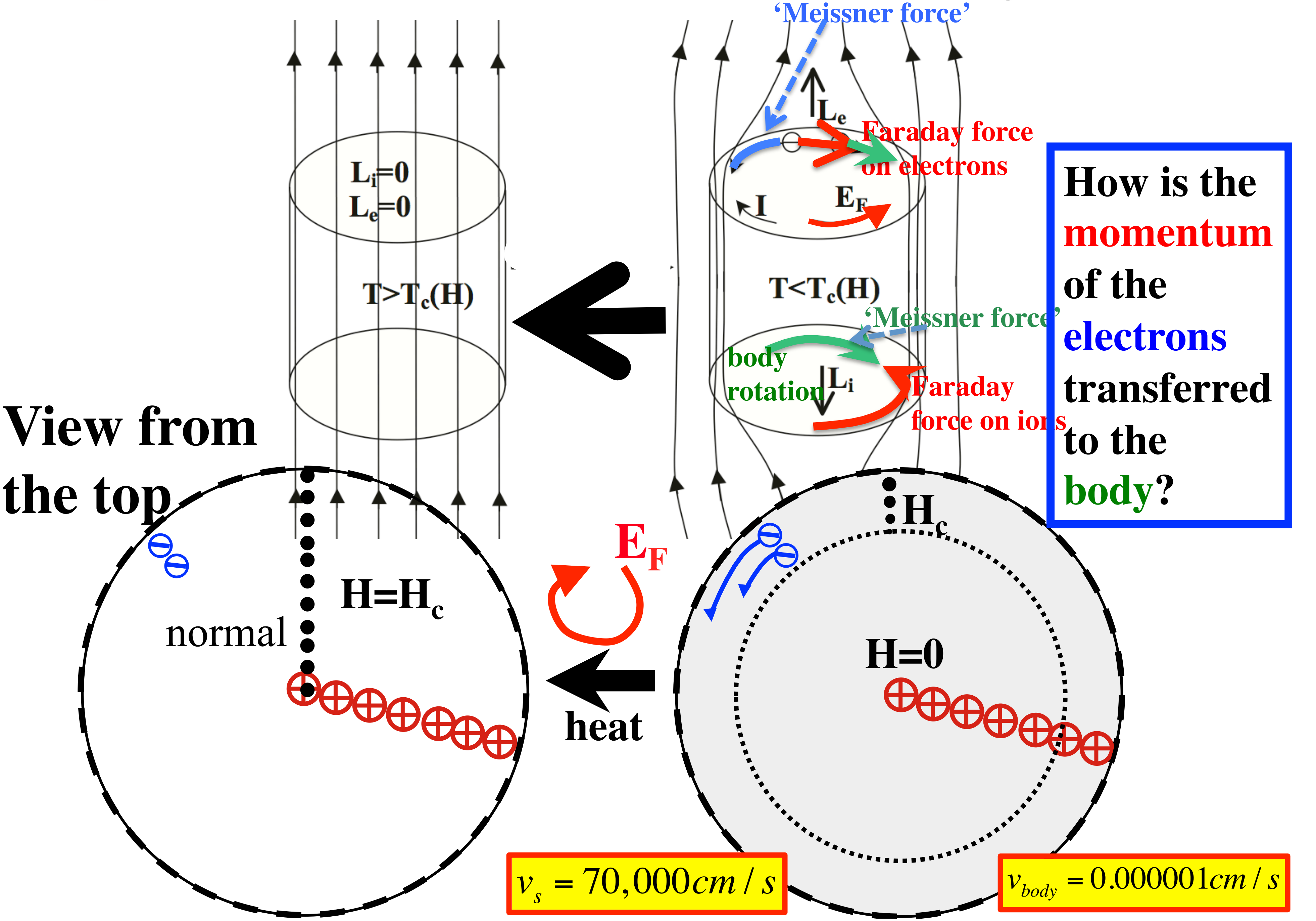


Normal to superconductor transition in a magnetic field



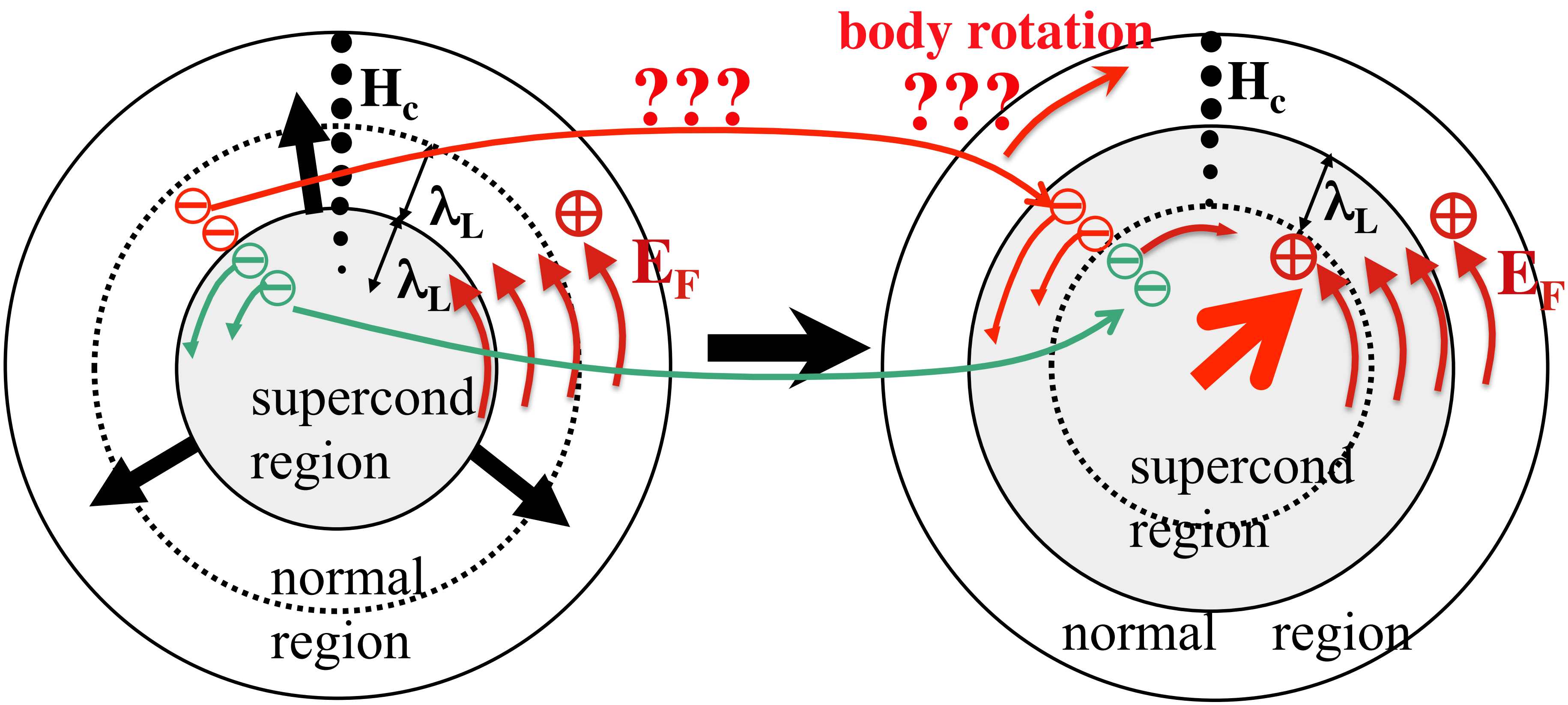
How is the momentum of the electrons and body generated?

Superconductor to normal transition in a magnetic field



How is the **momentum** of the **electrons** transferred to the **body**?

Expansion of superconducting phase



E_F slows down electrons inside superconducting (S) region

E_F imparts counterclockwise momentum to body in S region

WHAT IS NEEDED TO EXPLAIN THE PROCESS

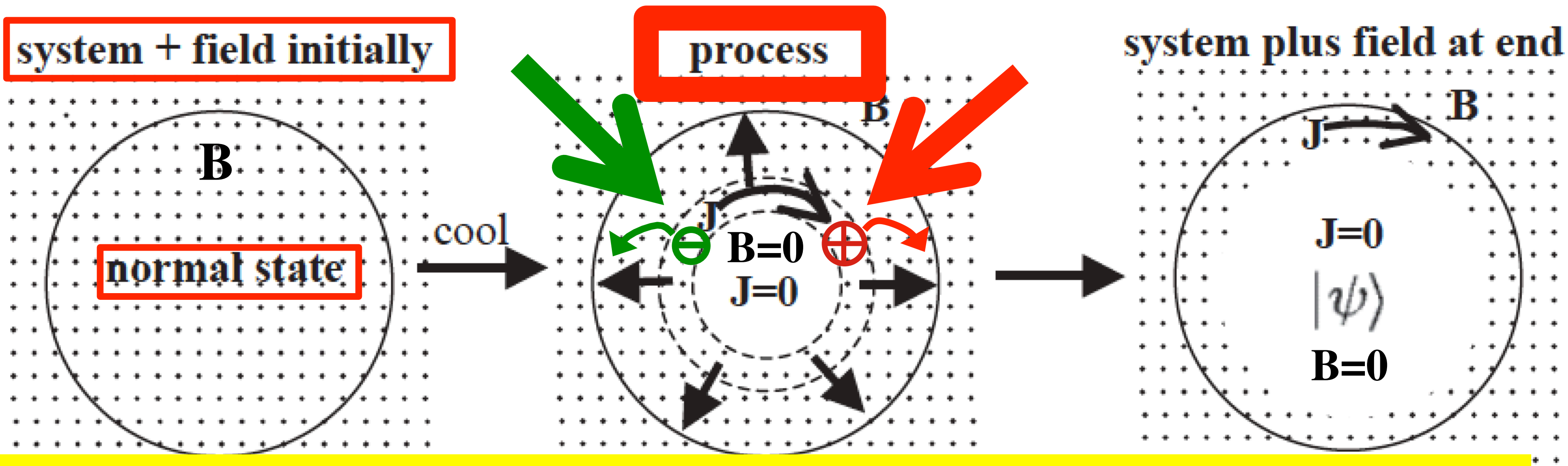
1) Electrons flowing outward

2) Ions flowing outward

3) Lorentz force

$$F_L = q\frac{\vec{v}}{c} \times \vec{B}$$

WHAT THE MEISSNER EFFECT IS



BCS cannot explain the process of field expulsion

WHAT IS NEEDED TO EXPLAIN THE PROCESS

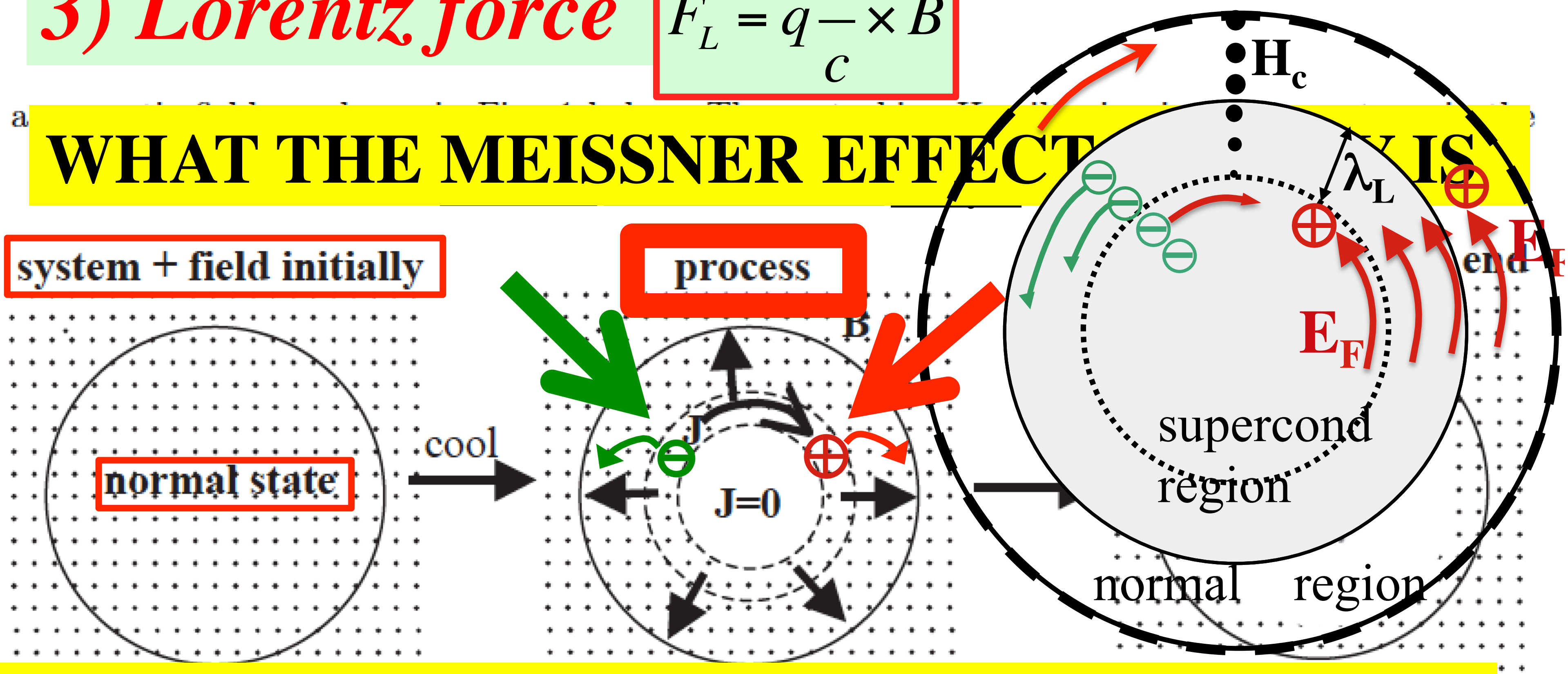
1) Electrons flowing outward

2) Ions flowing outward

3) Lorentz force $F_L = q\frac{\vec{v}}{c}\times\vec{B}$

radial flow of charge

WHAT THE MEISSNER EFFECT IS



BCS cannot explain the process of field expulsion

WHAT IS NEEDED TO EXPLAIN THE PROCESS

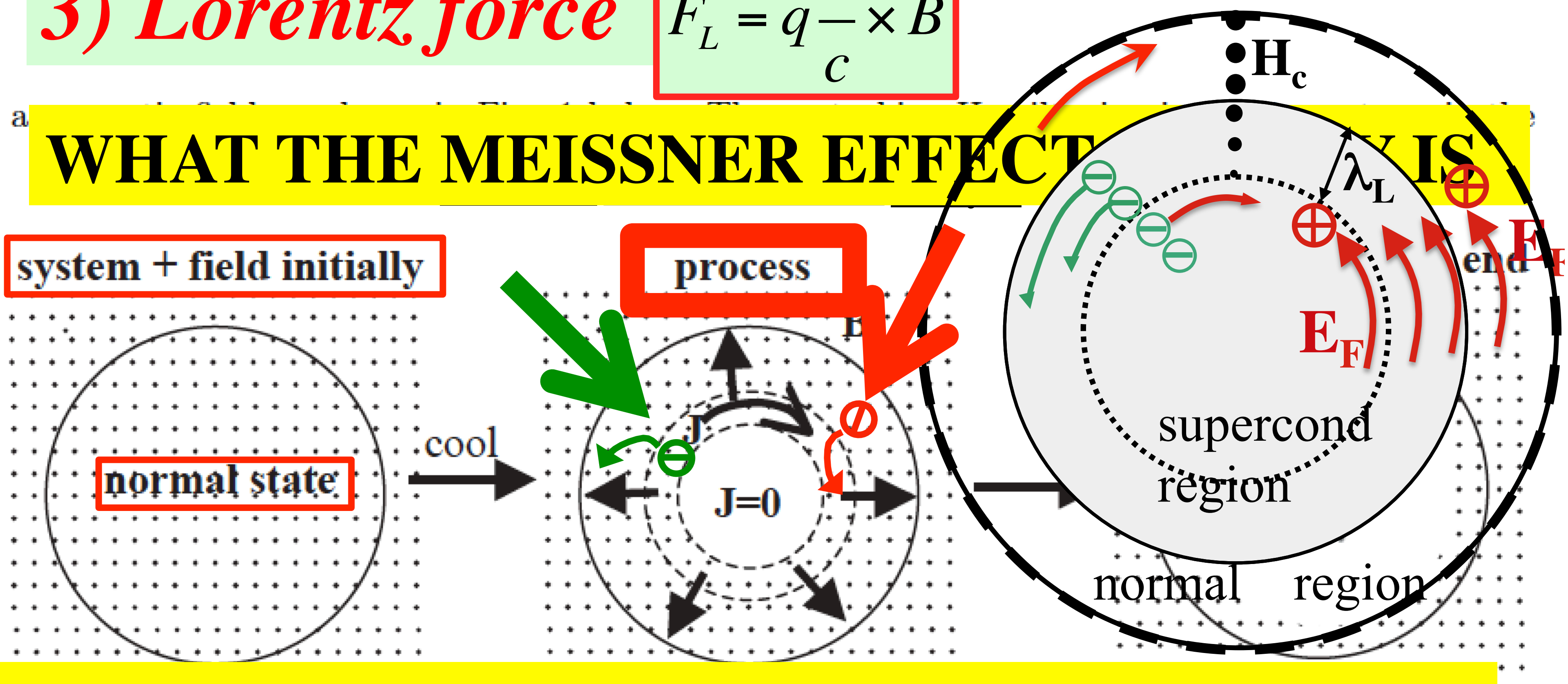
1) Electrons flowing outward

2) Electrons backflowing inward

radial flow of charge

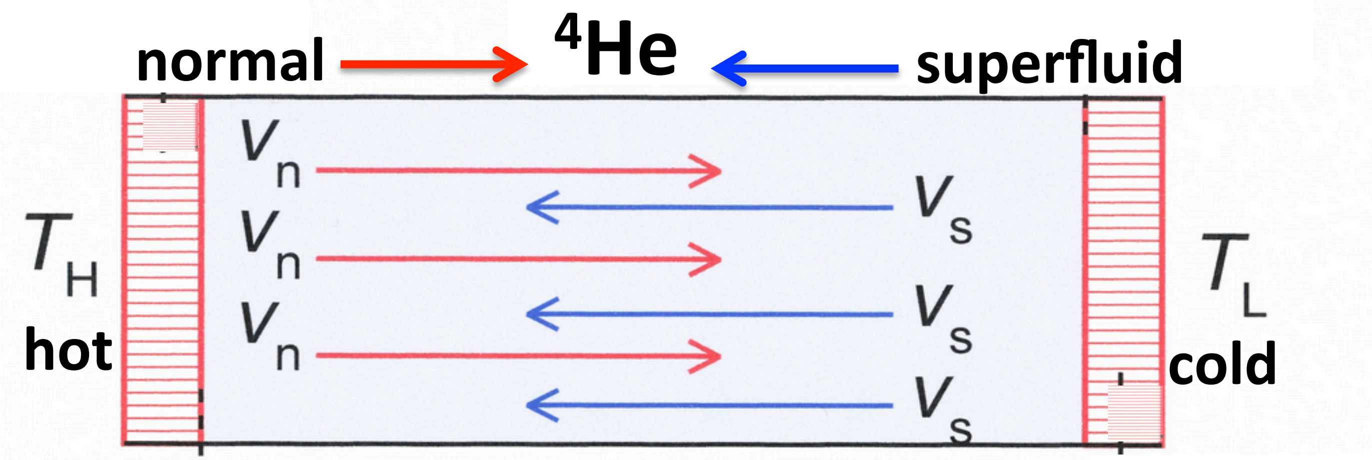
3) Lorentz force $F_L = q\frac{\vec{v}}{c} \times \vec{B}$

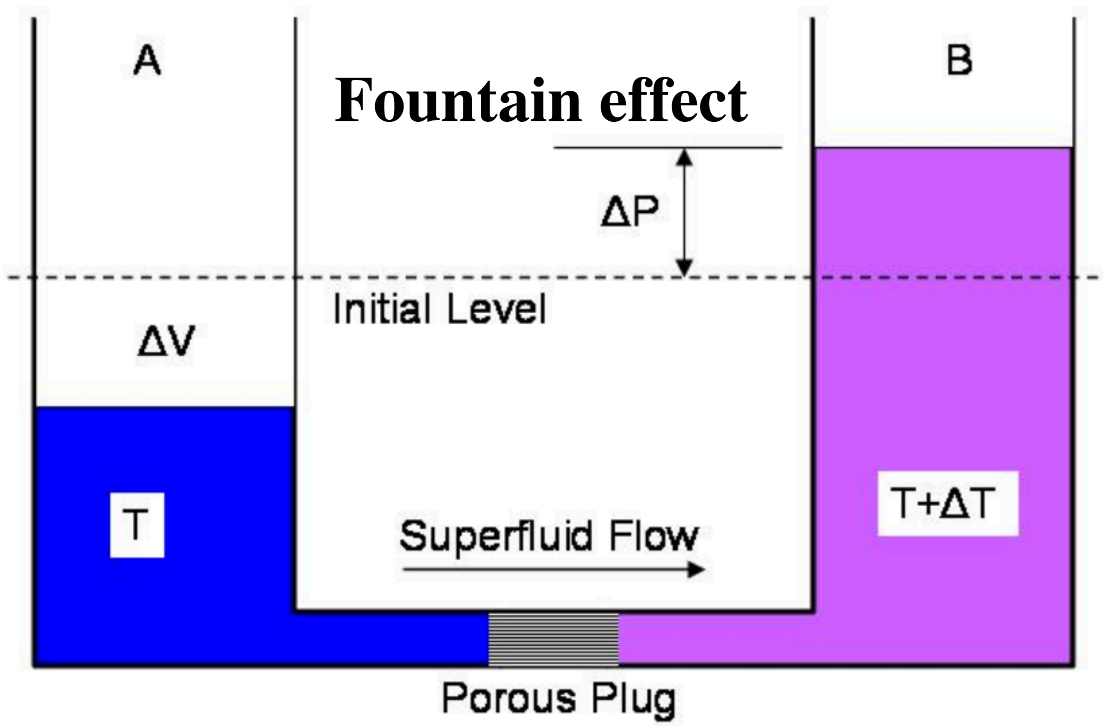
WHAT THE MEISSNER EFFECT IS



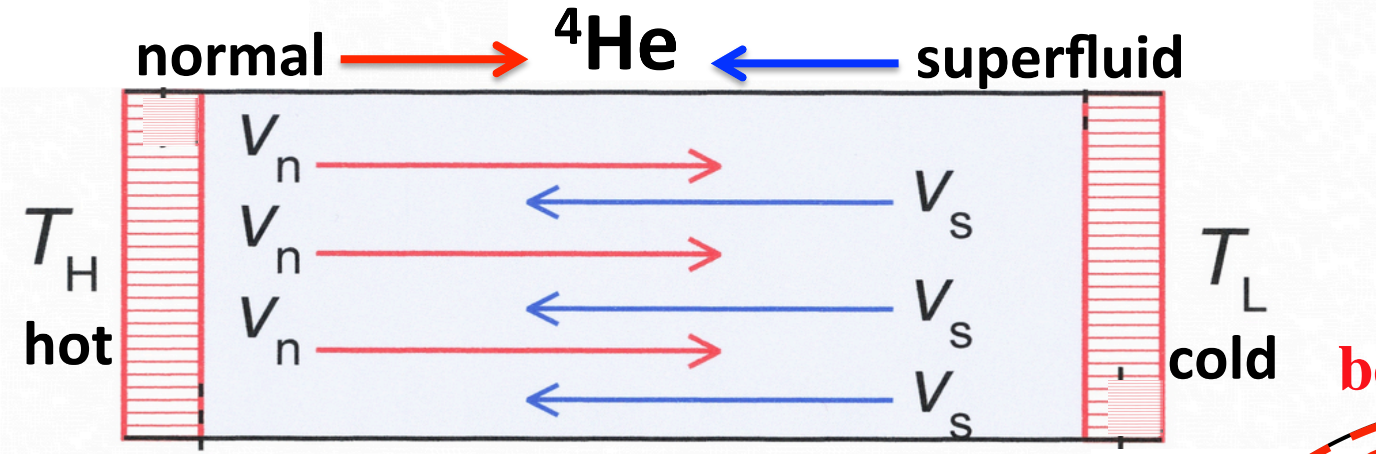
BCS cannot explain the process of field expulsion

Flow and backflow in superfluid ^{4}He



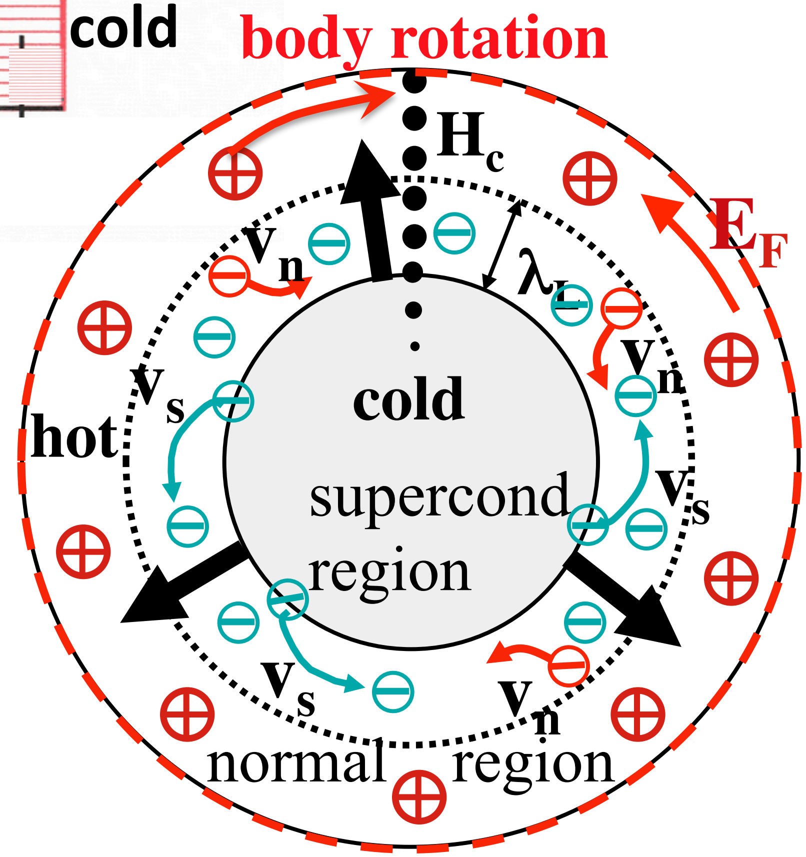


Flow and backflow in superfluid ^{4}He



Lorentz force:

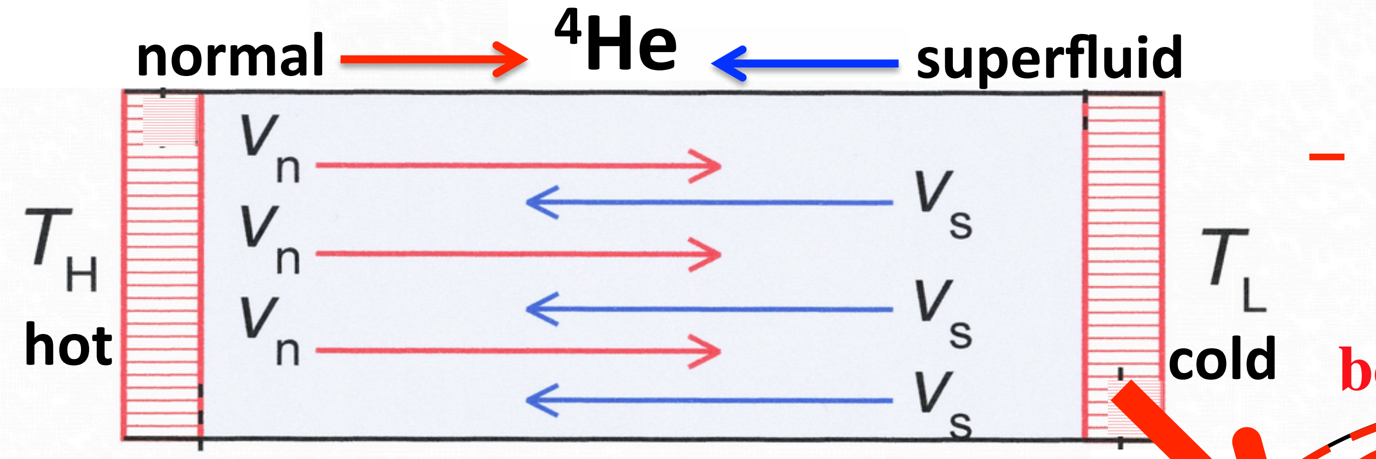
$$F_L = e\frac{\vec{v}}{c}\times\vec{B}$$



outflowing superelectrons generate Meissner current v_s

backflowing normal electrons v_n **transmit their momentum to the body. HOW?**

Flow and backflow in superfluid ^{4}He



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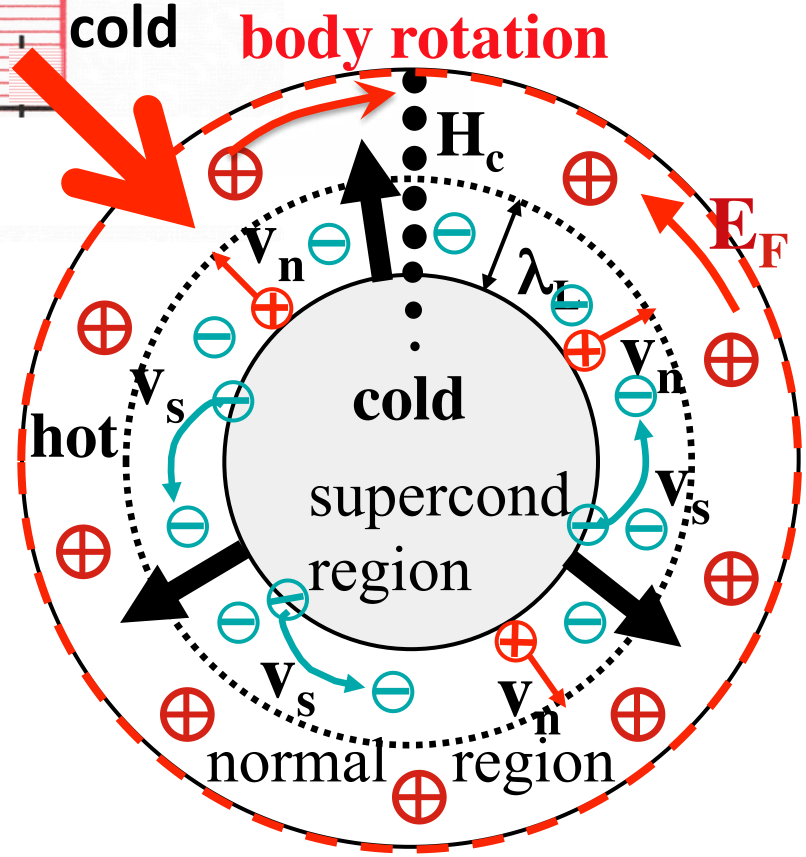
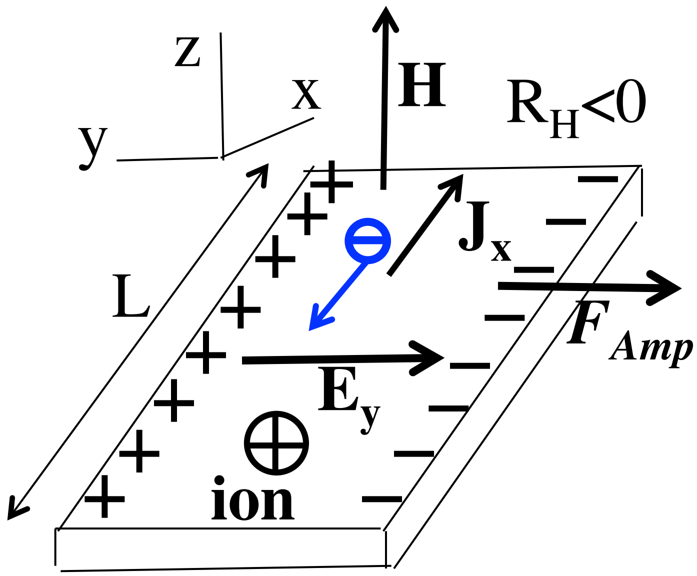


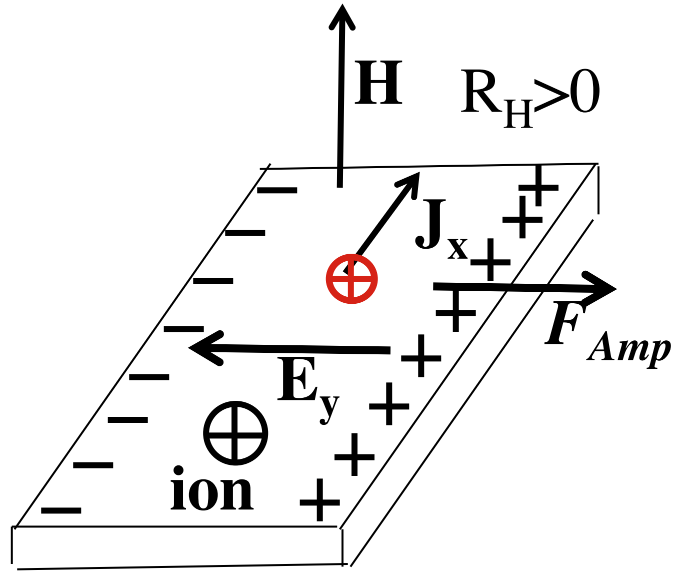
outflowing superelectrons generate Meissner current v_s

backflowing normal electrons v_n **transmit their momentum to the body. HOW?**

backflowing normal electrons = forward-flowing normal holes

What is the difference between electrons and holes?





$$\vec{F}_{Amp} = \frac{I}{c}\vec{L}\times\vec{H}$$

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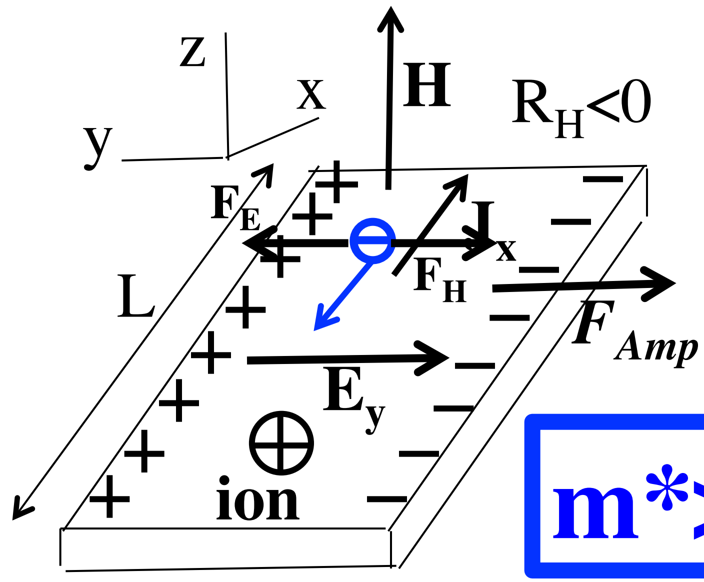
$$F_{Amp} = \oplus E_y$$

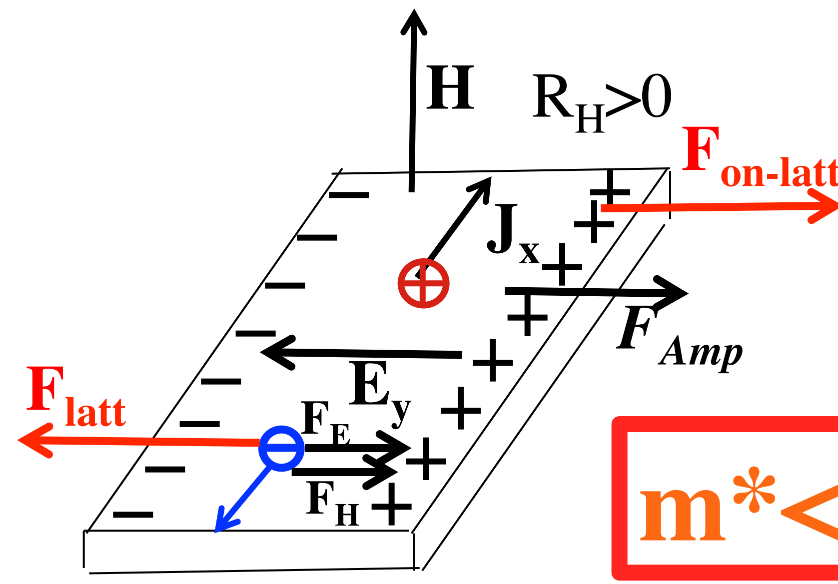
$$F_{Amp} \neq \oplus E_y$$

Ampere force on the body is caused by electric field E_y

What is the difference between electrons and holes?



m*>0



m*<0

$$\vec{F}_{Amp} = \frac{I}{c}\vec{L} \times \vec{H}$$

$$\vec{F}_{Amp} = \frac{I}{c}\vec{L} \times \vec{H}$$

$F_{Amp} = \oplus \, E_y$

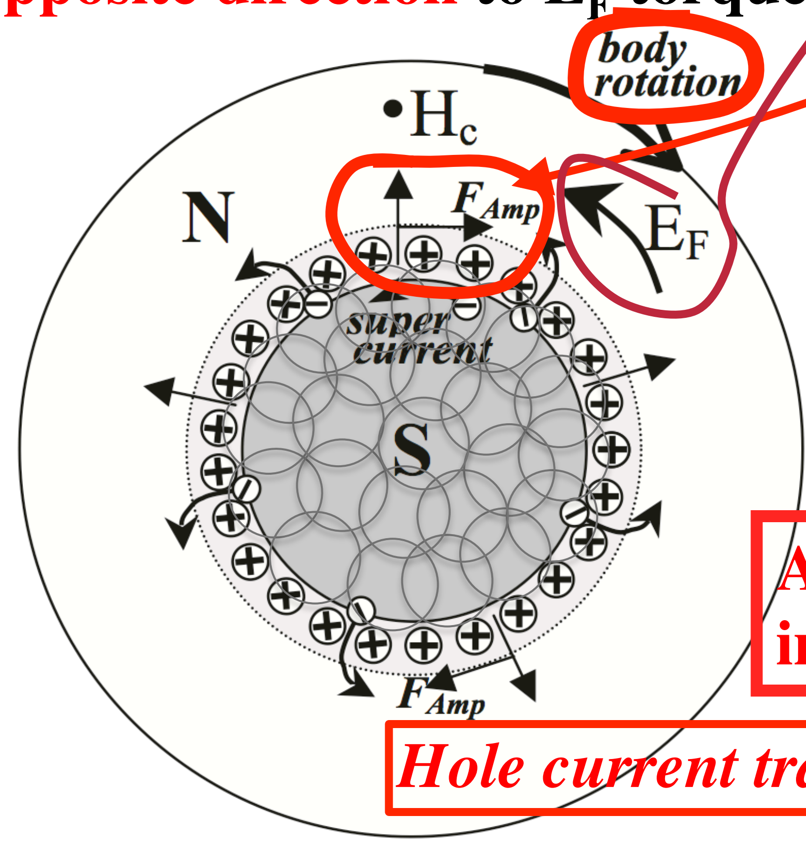
$F_{Amp} \neq \oplus \, E_y$

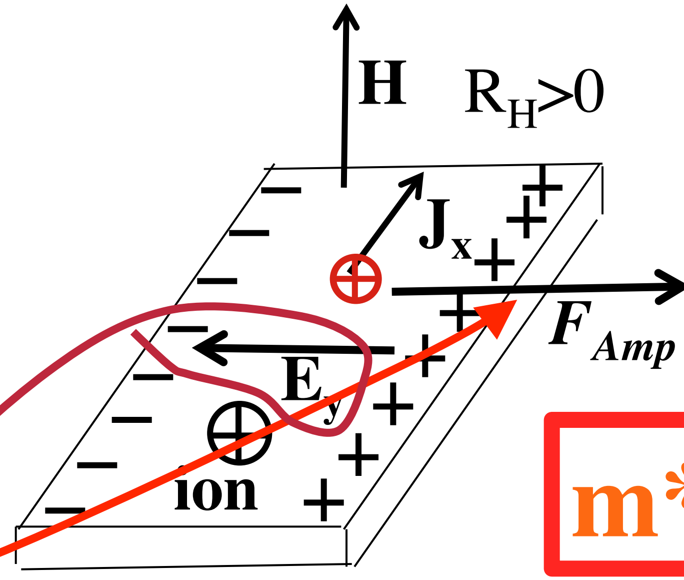
Ampere force on the body is caused by electric field E_y

Ampere force on the body is in direction <u>opposite</u> to electric field

Hole current transfers momentum to the body=F_{Amp}

There is an **outflowing hole current** causing an **Ampere torque** on body, in **opposite direction** to E_F torque





m*<0

$$\vec{F}_{Amp} = \frac{I}{c}\vec{L}\times\vec{H}$$

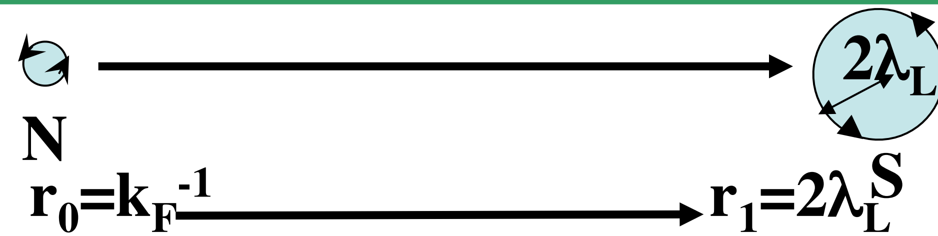
$$F_{Amp} \neq \oplus \leftarrow E_y$$

Ampere force on the body is in direction opposite to electric field

Hole current transfers momentum to the body=F_{Amp}

There is an outflowing electron current as normal electrons becoming superconducting expand their orbits, causing Meissner current

orbit expansion:

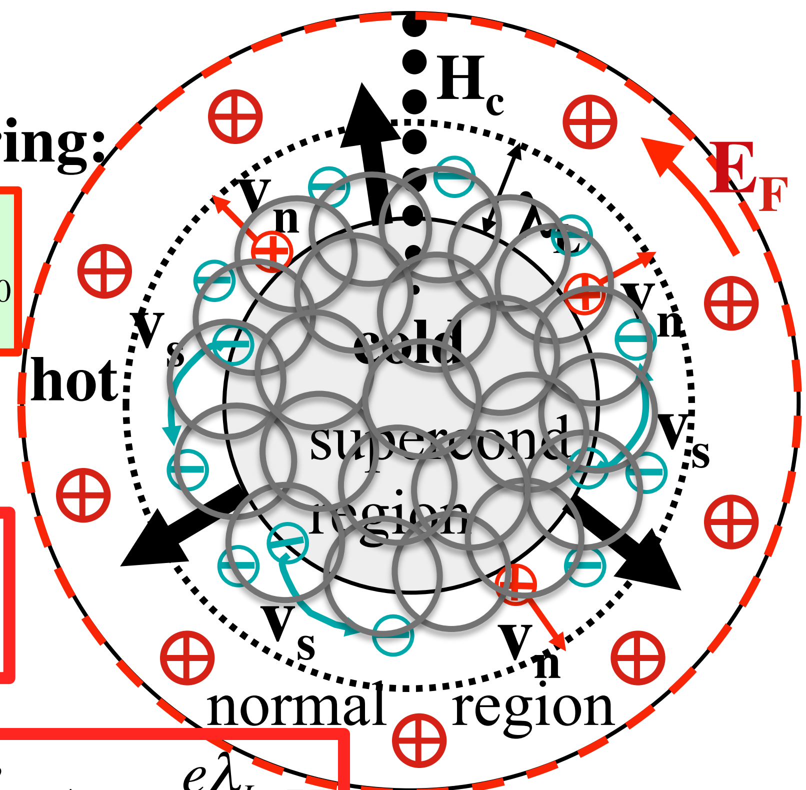


There is an outflowing hole current to preserve charge neutrality causing an Ampere torque on body, in opposite direction to E_F torque

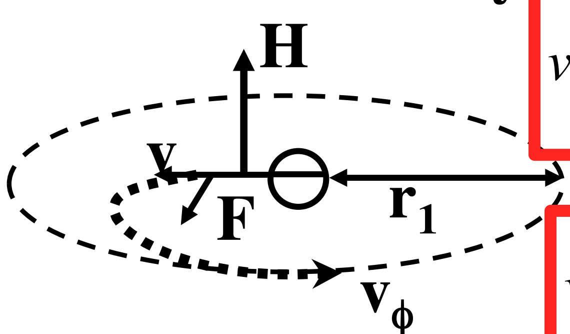
Orbit expansion ➔ kinetic energy lowering:

$$K_0 = \frac{1}{2} m_e v^2 = \frac{L^2}{2 m_e r_0^2} = \frac{\hbar^2}{2 m_e r_0^2}$$

$$K_1 = \frac{\hbar^2}{2 m_e r_1^2} << K_0$$



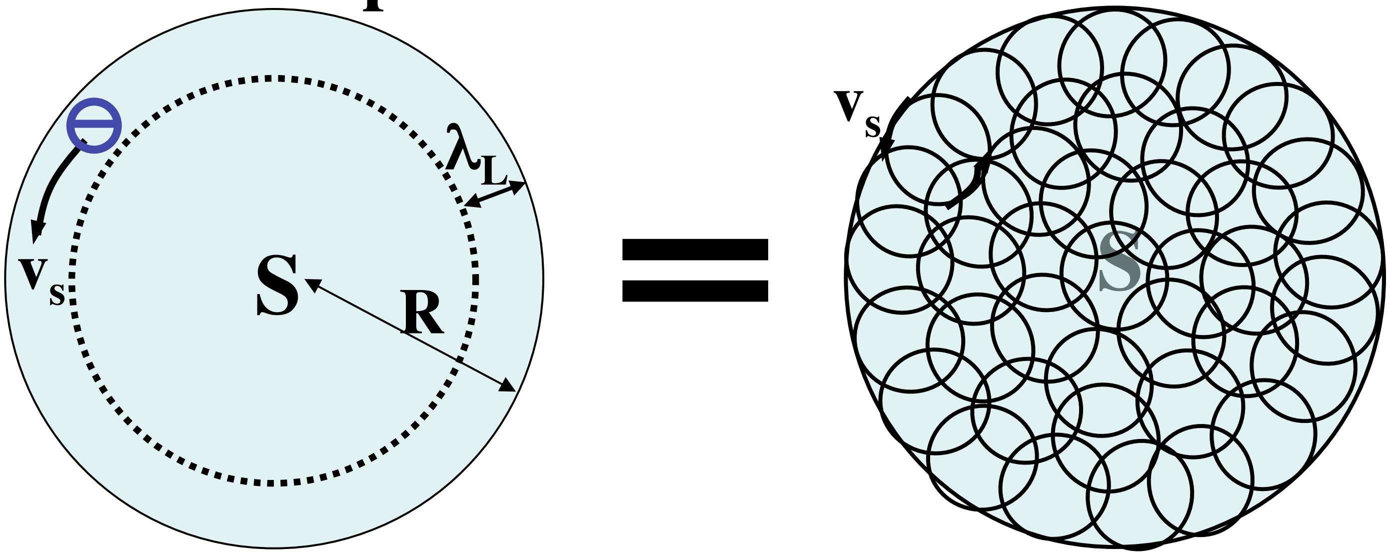
Orbit expanding in H field acquires azimuthal velocity

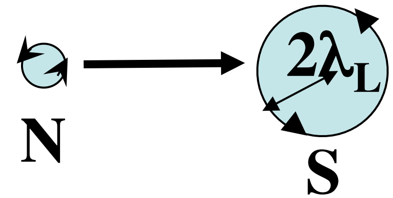


$$v_\phi = -\frac{e r_1}{2 m_e c} H = -\frac{e \lambda_L}{m_e c} H$$

$$v_s = \frac{1}{m_e}\left(p - \frac{e}{c} A\right) = -\frac{e}{m_e c} A = -\frac{e \lambda_L}{m_e c} H$$

$2\lambda_L$ orbits in superconductors





angular momentum of supercurrent:

$$L = (m_e v_s R) n_s (2\pi R \lambda_L h) \quad = \quad L = (m_e v_s 2\lambda_L) n_s (\pi R^2 h)$$

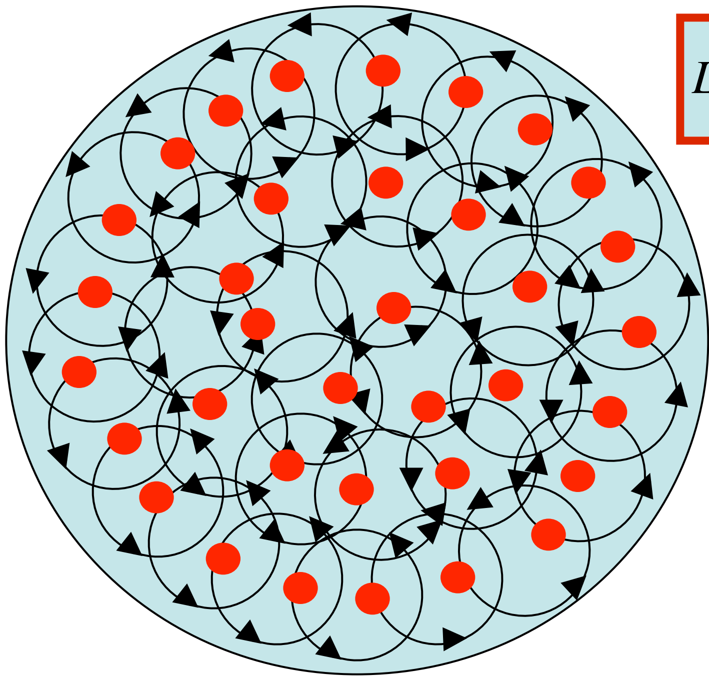
Ground state of a superconductor (no magnetic field applied)

$r=2\lambda_L$ orbits

$$v_\phi = \frac{\hbar}{4 m_e \lambda_L}$$

$$L = m_e v_\phi \cdot (2\lambda_L) = \frac{\hbar}{2}$$

$r=2\lambda_L$ orbits



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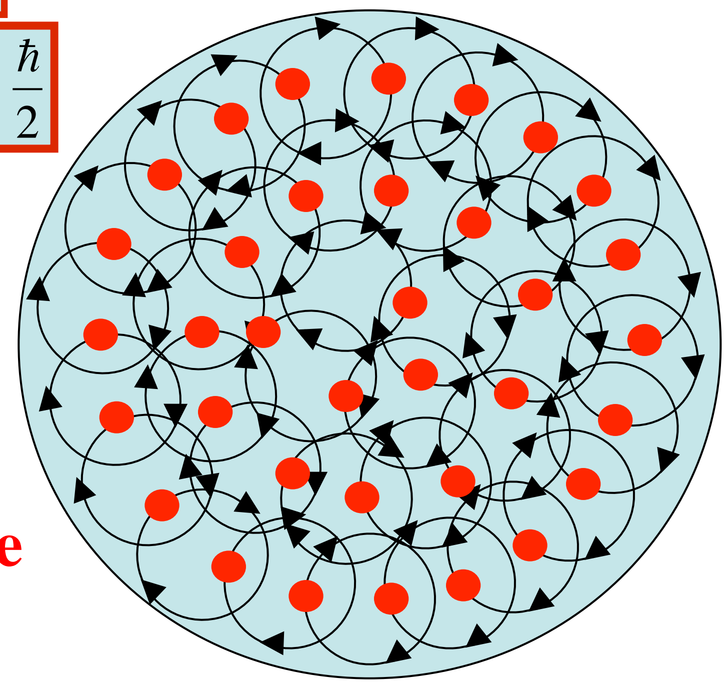
● **phase coherence**



Electron spin into screen

Electron spin out of screen

Macroscopic zero point motion in the ground state of superconductors

Currents in the interior cancel out, near the surface survive

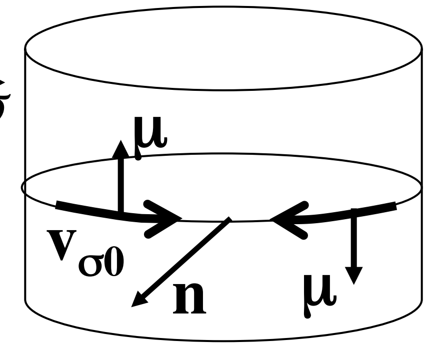
==> there is a spontaneous spin current in the ground state of superconductors near the surface

There is a spontaneous spin current in the ground state of superconductors, flowing within λ_L of the surface **Ann. der Phys.17, 380 (2008)**

$$\vec{v}_{\sigma 0} = -\frac{\hbar}{4m_e\lambda_L}\vec{\sigma}\times\hat{n}$$

no external fields applied

$$\vec{\mu} = \frac{e\hbar}{2m_e c}\vec{\sigma}$$



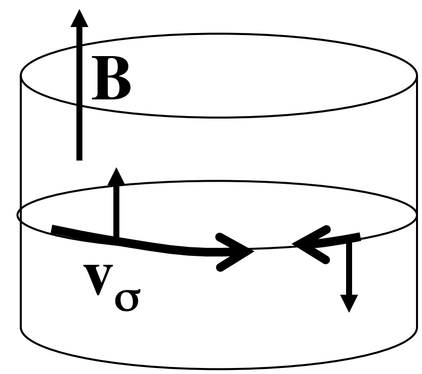
For λ_L=400A, $v_{\sigma 0}$=72,395cm/s

# of carriers in the spin current: n_s

When a magnetic field is applied:

$$\vec{v}_\sigma = \vec{v}_{\sigma 0} - \frac{e}{m_e c}\lambda_L \vec{B}\times\hat{n}$$

$$\vec{J}_\sigma = en_s\vec{v}_\sigma = \vec{J}_{\sigma 0} - \frac{c}{4\pi\lambda_L}\vec{B}\times\hat{n}$$



The slowed-down spin component stops when

$$B = \frac{m_e c}{e\lambda_L}v_{\sigma 0} = \frac{\hbar c}{4e\lambda_L^2} = \frac{\phi_0}{4\pi\lambda_L^2} \sim H_{c1}!$$

Electronic orbits have $2\lambda_L$ radius (to explain Meissner effect)

Angular momentum: $L = m_e v_{\sigma 0}(2\lambda_L)$ ==> $L = \hbar/2$

There is a spontaneous spin current in the ground state of superconductors, flowing within λ_L of the surface **Ann. der Phys.17, 380 (2008)**

$$\vec{v}_{\sigma 0} = -\frac{\hbar}{4m_e\lambda_L}\vec{\sigma}\times\hat{n}$$

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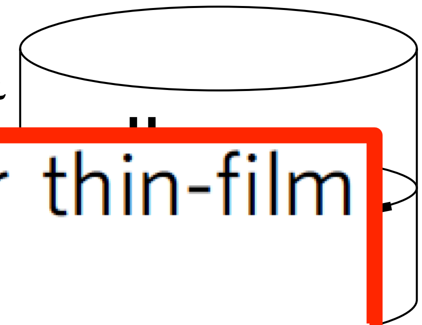


Universal self-field critical current for thin-film superconductors

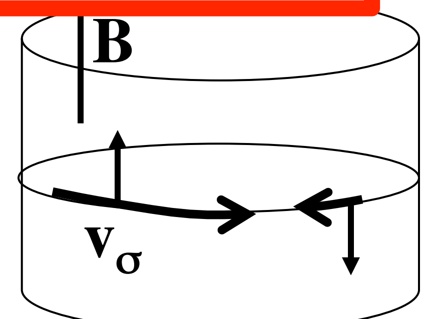
E.F. Talantsev[1] & J.L. Tallon[1,2]

Nature Communications **6**, Article number: 7820 (2015)

+ other papers

$$\vec{v}_{\sigma} = \vec{v}_{\sigma 0} - \frac{e}{m_ec}\lambda_L\vec{B}\times\hat{n}$$

$$\vec{J}_{\sigma} = en_s\vec{v}_{\sigma} = \vec{J}_{\sigma 0} - \frac{c}{4\pi\lambda_L}\vec{B}\times\hat{n}$$



The slowed-down spin component stops when

$$B = \frac{m_ec}{e\lambda_L}v_{\sigma 0} = \frac{\hbar c}{4e\lambda_L^2} = \frac{\phi_0}{4\pi\lambda_L^2} \sim H_{c1}!$$

$$\vec{J}_c = -\frac{c}{4\pi\lambda_L}H_{c1}\times\hat{n}$$

Electronic orbits have $2\lambda_L$ radius (to explain Meissner effect)

Angular momentum: $L = m_e v_{\sigma 0}(2\lambda_L)$ **==>** $L = \hbar/2$

Why electronic orbits expand to radius $2\lambda_L$ when a metal becomes superconducting:

From Dirac equation: $H_{s.o.} = -\frac{e\hbar}{4m_e^2c^2}\vec{\sigma}\cdot(\vec{E}\times\vec{p})$ **spin-orbit interaction**

EPL, 81 (2008) 67003

$$H = \frac{p^2}{2m_e} + H_{s.o.} = \frac{1}{2m_e}(\vec{p} - \frac{e}{c}\vec{A}_\sigma)^2$$

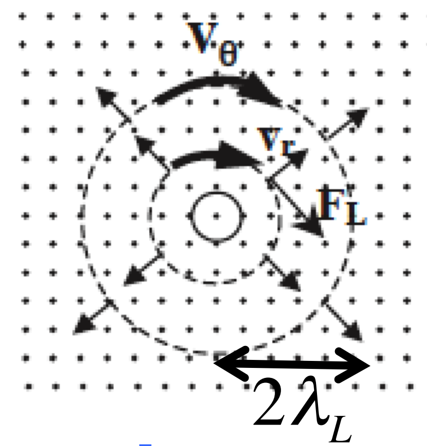
$$\vec{p} = 0 \quad \Rightarrow \quad v_\sigma^0 = -\frac{e}{m_e c}\vec{A}_\sigma$$

$$\vec{A}_\sigma = \frac{\hbar}{4m_e c}\vec{\sigma}\times\vec{E}$$

$$\vec{E} = 2\pi\rho\vec{r} = 2\pi|e|n_s\vec{r}$$

$$\vec{A}_\sigma = \frac{2\pi|e|n_s\hbar}{4m_e c}\vec{\sigma}\times\vec{r} \equiv \frac{\vec{B}_\sigma\times\vec{r}}{2}$$

$$\Rightarrow \vec{B}_\sigma = \frac{\pi|e|n_s\hbar}{m_e c}$$



$$v_\sigma^0 = \frac{e\lambda_L}{m_e c}B_\sigma = \frac{\pi e^2 n_s e^2\lambda_L\hbar}{m_e^2c^2} = \frac{\hbar}{4m_e\lambda_L}$$

angular momentum

$$\ell = m_e v_\sigma^0(2\lambda_L) = m_e\frac{\hbar}{4m_e\lambda_L}(2\lambda_L) = \frac{\hbar}{2}$$

$$\ell = \frac{\hbar}{2}$$

EPL, 113 (2016) 37001

Theory of hole superconductivity (1988-2018)

F. Marsiglio

References: http://physics.ucsd.edu/~jorge/hole.html

S. Tang, H.Q. Hong
H.Q. Lin R. Teshima
G. Bach

HOLE SUPERCONDUCTIVITY (1989)

Hole superconductivity and the high-T_c oxides (1990)

Superconductivity in the transition-metal series (1992)

Hole superconductivity in MgB_2: a high T_c cuprate without Cu (2001)

Hole superconductivity in arsenic–iron compounds (2008)

Why non-superconducting metallic elements become superconducting under high pressure (2010)

Hole superconductivity in H_2S and other sulfides under high pressure (2015)

Materials

TUNNELING ASYMMETRY: A TEST OF SUPERCONDUCTIVITY MECHANISMS

Superconductors that change color when they become superconducting

Optical sum rule violation, superfluid weight, and condensation energy in the cuprates

(1989-2000)

Dynamic Hubbard Model

HOLE SUPERCONDUCTIVITY FROM KINETIC ENERGY GAIN

Charge expulsion, charge inhomogeneity, and phase separation in dynamic Hubbard models

(1989-on)

Charge expulsion and electric field in superconductors

Spin currents in superconductors

Electrodynamics of spin currents in superconductors

(2001-2008)

On the reversibility of the Meissner effect and the angular momentum puzzle

Momentum of superconducting electrons and the explanation of the Meissner effect

Why only hole conductors can be superconductors

(2016-18)

Prediction vs postdiction

The essential elements for explaining Meissner effect

1) Electrons expelled radially outward (2001)
(to generate Meissner current)

2) Holes are the normal charge carriers (1989)
(to transfer momentum to body without dissipation)

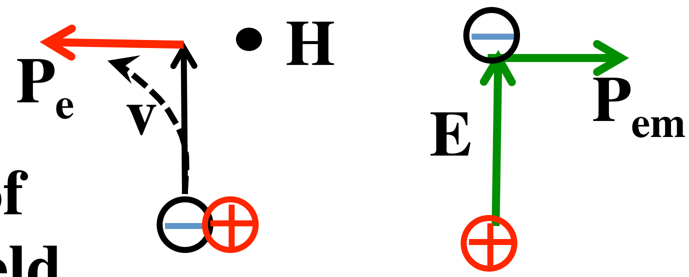
3) Kinetic energy lowering drives superconductivity
(to explain what drives electron outflow) (1992)

were part of the theory many years before the theory addressed the Meissner effect (2003-on)

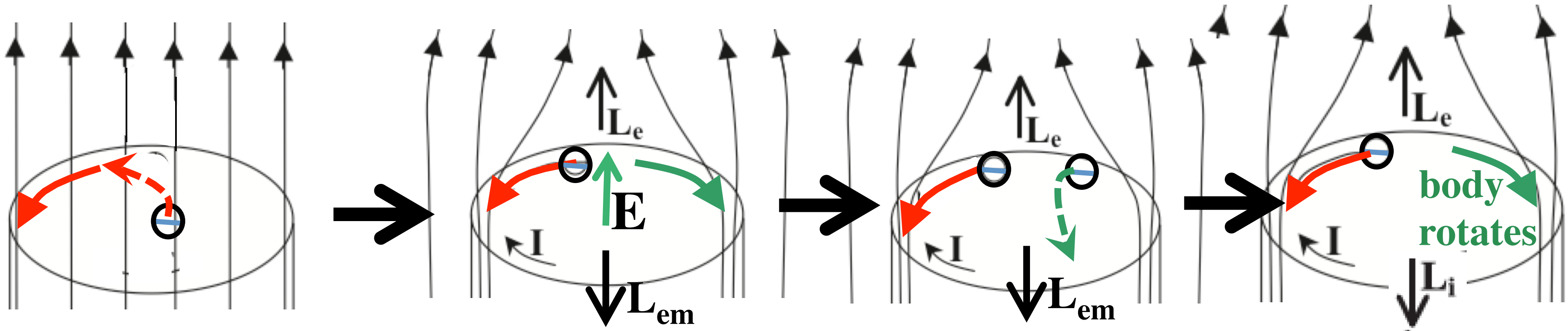
How momentum is transferred without transferring energy

$$\vec{P}_{em} = \frac{1}{4\pi c}\vec{E}\times\vec{H}$$

P_{em}= momentum of electromagnetic field



$$\vec{F}_H = e\frac{\vec{v}}{c}\times\vec{H}$$



electron moves out, acquires P_e

em field acquires P_{em}

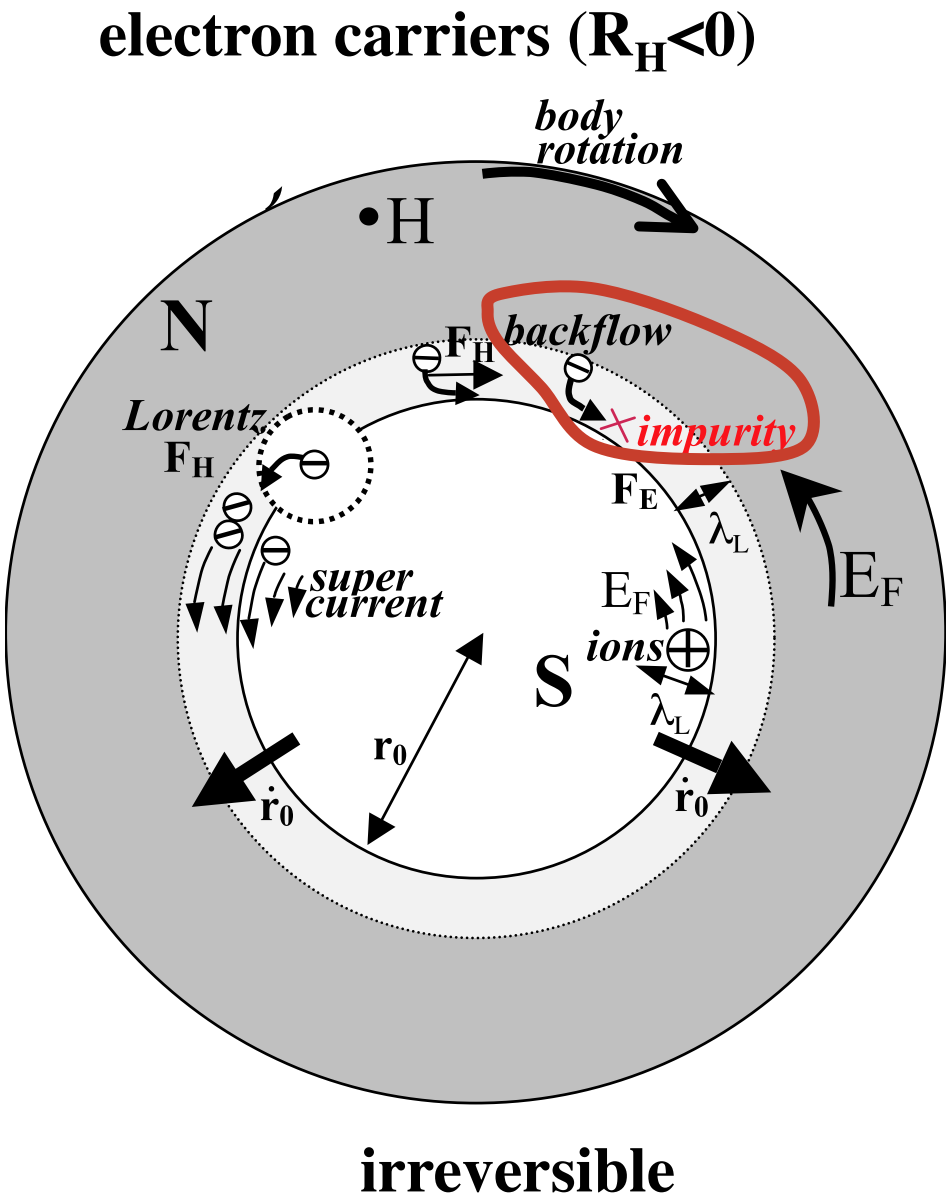
normal electron moves in, acquires P_{em} from em field, transfers it to the body

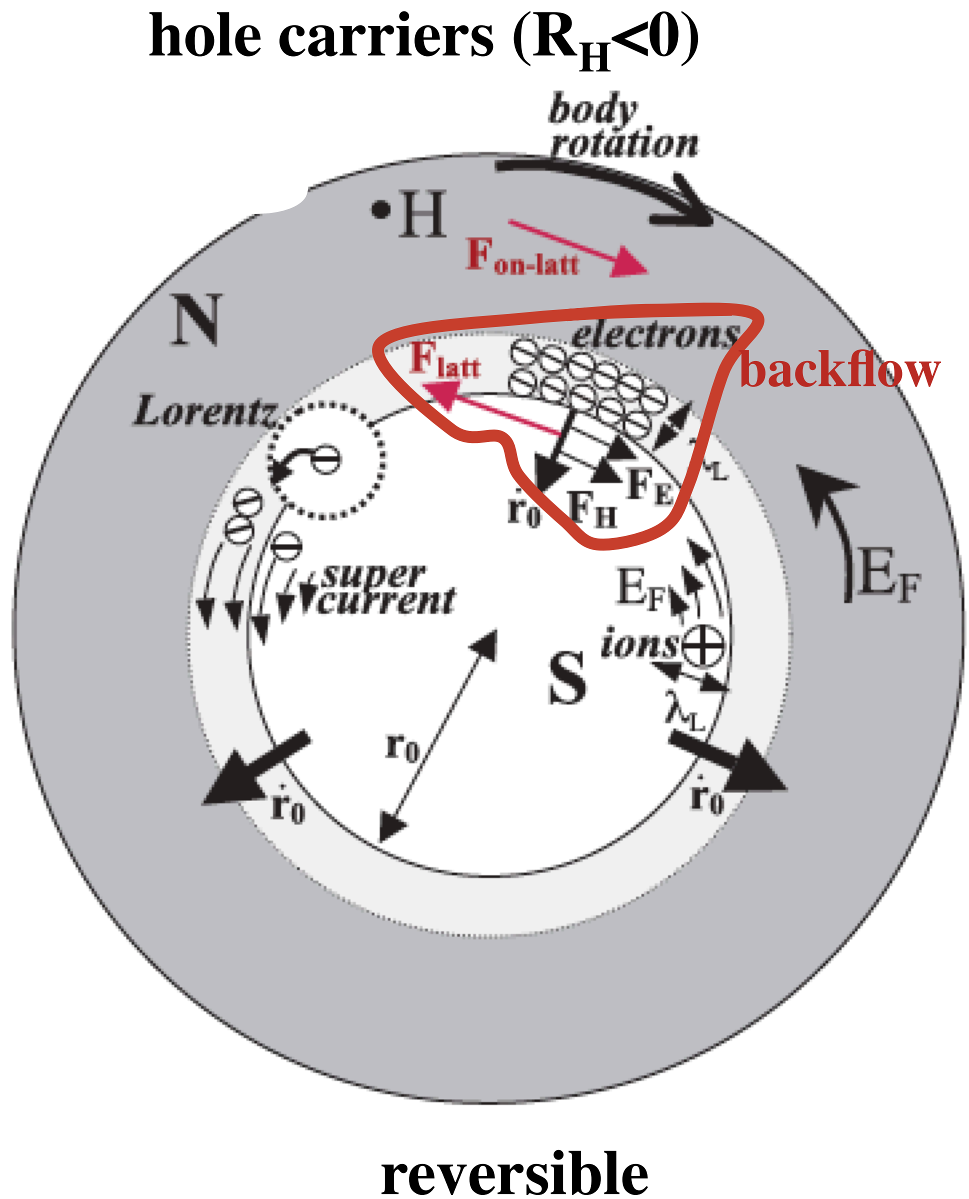
Radial flow and counterflow is essential

Electromagnetic field mediates the transfer of momentum

Process is reversible iff the normal state charge carriers are holes

Why hole carriers are necessary for the Meissner effect





Summary: In the Meissner transition,

* Electrons need to acquire momentum in direction opposite to that dictated by the Faraday electric field.
* The body needs to acquire momentum in direction opposite to that dictated by the Faraday electric field.
* The transition is thermodynamically reversible

==> A radial outflow of superconducting electrons is needed, and a radial outflow of normal holes

Holes are necessary to transfer momentum from electrons to the body in a reversible way

* $2\lambda_L$ orbits describe radial outflow, give spin current, explain universal self-field critical current

* Conventional BCS-London theory describes NONE of this
* The theory of hole superconductivity describes this physics

References: http://physics.ucsd.edu/~jorge/hole.html

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