Inconsistency of the conventional theory of superconductivity

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In a process where the temperature of a type I superconductor in a magnetic field changes, the conventional theory of superconductivity predicts that Joule heat is generated and that the final state is independent of the speed of the process. I show that these two predictions cannot be simultaneously reconciled with the laws of thermodynamics. I propose a resolution of this paradox.

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I. INTRODUCTION

Within the conventional London-BCS theory of superconductivity [1], the state of a simply connected superconductor in an external magnetic field is independent of how the system reached that state. The theory also predicts that when an electric field exists in a superconductor at finite temperature, Joule heat is always generated. Here I point out that for a type I superconductor in the presence of a magnetic field, those two assumptions lead to a contradiction with the laws of thermodynamics in a process where the temperature is changed below T_c. Consequently, one of the two assumptions must be incorrect in that situation. I propose it is the second one, and that the alternative theory of hole superconductivity offers a possible resolution of this paradox.

II. THE PROCESS

Figure 1 shows the phase diagram of a type I superconductor in a magnetic field H [1]. We consider the process where a cylindrical superconductor is cooled from state 1 to state 2 shown in Fig. 1, in the presence of an applied field H_0. The inconsistency also arises if we consider heating instead. The magnetic field of a long cylinder of radius R and London penetration depth λ_L in a magnetic field H_0 parallel to its axis is [2]

$$\vec{B}(r) = H_0 \frac{J_0(ir/\lambda_L)}{J_0(iR/\lambda_L)} \hat{z} \tag{1}$$

where J_0 is the Bessel function of order 0 and $\hat{z}$ is along the cylinder axis. To lowest order in λ_L/R,

$$\vec{B}(r) = H_0 e^{(r-R)/\lambda_L} \hat{z}. \tag{2}$$

The London penetration depth is a decreasing function of temperature, hence a decreasing function of time in the process of cooling. In the process shown in Fig. 1, the London penetration depth changes from λ_1 to $\lambda_2 < \lambda_1$ when the temperature is lowered from T_1 to T_2. Figure 2 shows the superconductor as seen from the top, with the dots indicating magnetic field pointing out of the paper.

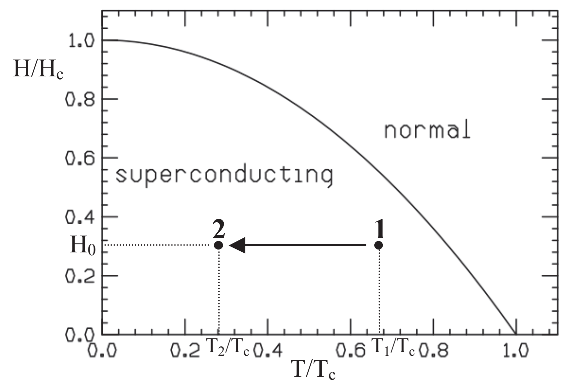

FIG. 1: Critical magnetic field versus temperature for a type I superconductor. We will consider the process where a system evolves from point 1 to point 2 along the direction of the arrow.

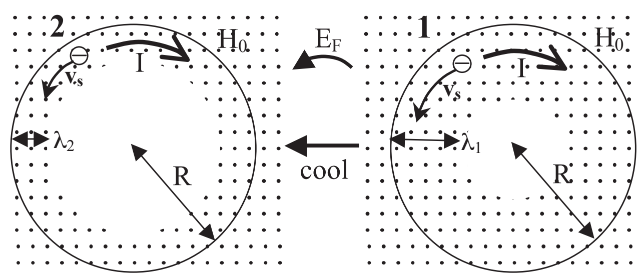

FIG. 2: Cylindrical superconductor seen from the top. The right (left) panel indicates the system in the state 1 (2) of Fig. 1. The dots indicate magnetic field H_0 coming out of the paper. The same current I flows in both states. The Faraday electric field E_F generated during the process points counterclockwise.

III. FARADAY ELECTRIC FIELD

The magnetic field near the surface is changing in this process, therefore a Faraday electric field is generated. We assume cylindrical symmetry throughout the process. The electric field at radius r at time t is determined by the equation

$$\oint \vec{E}(r,t) \cdot d\vec{\ell} = -\frac{1}{c}\frac{\partial}{\partial t}\int_{r'<r} \vec{B}(r',t) \cdot d\vec{S} \tag{3}$$

which yields

$$\vec{E}(r,t) = -\frac{H_0}{c}(1+\frac{R-r}{\lambda_L})e^{(r-R)/\lambda_L(t)}\frac{\partial\lambda_L}{\partial t}\hat{\theta}. \quad (4)$$

The electric field points counterclockwise.

At any given temperature there are both superfluid and normal electrons, of density n_s and n_n, with $n_s + n_n = n$ constant in time, in a two-fluid description [1]. Similarly within BCS theory there is the superfluid and Bogoliubov quasiparticles at finite temperature, we will call the latter 'normal electrons' [1]. The Faraday electric field will impart momentum to these normal electrons during the process, and this momentum will decay to zero through scattering with impurities or phonons [1]. These are irreversible processes, that generate Joule heat and entropy [3]. The normal current induced by the Faraday electric field is

$$j_n(r,t) = \sigma_n(t)E(r,t) \quad (5)$$

with [1]

$$\sigma_n(t) = \frac{n_n(t)e^2\tau}{m^*} \quad (6)$$

within a Drude description with relaxation time τ, with m^* the transport effective mass. The energy dissipated per unit time per unit volume is

$$\frac{\partial w(r,t)}{\partial t} = \sigma_n(t)E(r,t)^2, \quad (7)$$

and the energy per unit time dissipated over the entire volume is

$$\frac{\partial W(t)}{\partial t} = \int d^3r\frac{\partial w(r,t)}{\partial t}. \quad (8)$$

If the process extends from time $t = 0$ to $t = t_0$ the total Joule heat dissipated is

$$Q_J = \int_0^{t_0}\frac{\partial W(t)}{\partial t}dt \quad (9)$$

and the Joule entropy generated during this process is

$$S_J = \int_0^{t_0}\frac{\partial W}{\partial t}\frac{1}{T(t)}dt \quad (10)$$

where $T(t)$ is the temperature at time t. We assume the process is sufficiently slow that $T(t)$ is well defined at all times.

Note that Q_J and S_J depend on the speed of the process. If we assume for simplicity that $\partial\lambda_L/\partial t$ is constant, we have

$$\int_0^{t_0}(\frac{\partial\lambda_L}{\partial t})^2F(\lambda_L(t))dt = \frac{\partial\lambda_L}{\partial t}\int_{\lambda_1}^{\lambda_2}F(\lambda_L)d\lambda_L \quad (11)$$

for any F, so Q_J and S_J are directly proportional to $\partial\lambda_L/\partial t$. In addition, Q_J and S_J are proportional to the Drude relaxation time τ, or equivalently to the normal state conductivity.

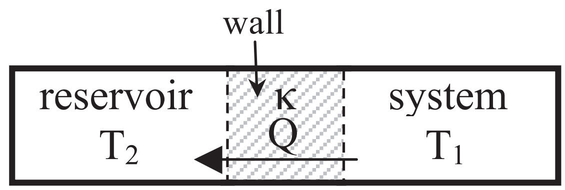

FIG. 3: The system (superconductor in a magnetic field) at initial temperature T_1 is connected to a heat reservoir at temperature $T_2 < T_1$ through a wall of thermal conductivity κ. The entire assembly is thermally and mechanically insulated from its environment.

IV. THERMODYNAMICS

We consider the situation shown in Fig. 3. The system is our superconductor with phase diagram given in figure 1, with applied magnetic field H_0. The system is initially in thermal equilibrium at temperature T_1, with London penetration depth $\lambda_1 = \lambda_L(T_1)$.

We put it in thermal contact with a heat reservoir at temperature $T_2 < T_1$ through a wall with thermal conductivity κ. Heat will flow and eventually the system will reach temperature T_2 and be in thermal equilibrium with the heat reservoir. We assume the entire assembly is thermally and mechanically insulated from its environment. The magnetic field originates in external permanent magnets, no work is performed on those magnets during the process. We also assume the process is sufficiently slow that no electromagnetic radiation is generated. Under these conditions, *the initial and final states of BOTH the system AND the reservoir are uniquely determined.*

Given the initial and final states, we can compute various thermodynamic quantities. The total heat Q transferred from the system to the reservoir during the process is

$$Q = \int_{T_2}^{T_1} dTC(T) \quad (12)$$

where $C(T)$ is the equilibrium heat capacity of our system. The change in entropy of the system in this process is

$$\Delta S = S(T_2) - S(T_1) = \int_{T_1}^{T_2} dT\frac{C(T)}{T} \quad (13)$$

and is of course negative since $T_2 < T_1$. The change in entropy of the universe in this process is

$$\Delta S_{univ} = \frac{Q}{T_2} + \Delta S \quad (14)$$

and is of course positive since we are dealing with an irreversible process, heat conduction between systems at different temperatures. The quantities Q and ΔS_{univ} depend *only* on the initial and final states of the process, *not* on the speed at which the process happens.

The Joule heat Q_J and associated entropy S_J discussed in the previous section depend on the speed of the process, which will depend principally on the thermal conductivity of the heat conductor, κ, connecting the system and the heat reservoir. It would appear that the existence of Joule heat violates both the first and second law of thermodynamics.

In the next section we will show that even if it may be possible to 'save' the first law by some contrived assumption, the second law is necessarily violated.

V. THE INCONSISTENCY

Let us consider a small step in the process, starting with the system at temperature T, where the system supplies heat ΔQ to the reservoir which is at temperature $T_2 < T - \Delta T$. The system will change its temperature from T to $T - \Delta T$. Assume we connect and disconnect the thermal connection between system and reservoir at the beginning and the end of this step, and wait at the end until equilibrium has been attained. Consider two different ways to do this step:

(a) Infinitely slowly

(b) In a finite amount of time, Δt.

According to the previous discussion, for (b) finite Joule heat will be generated in the system during this process.

First, let's realize that the change in temperature of the system, ΔT, has to be the same for (a) and (b). The reason is, the energy transferred to the reservoir was ΔQ for both (a) and (b), and the final state and hence final temperature of the system is uniquely determined by its energy, which is the same for (a) and (b), namely (initial energy -ΔQ). Note also that the energy in the electromagnetic field is also the same in the final state of processes (a) and (b). We discuss the electromagnetic field in detail elsewhere [4], it is not necessary to include it for this argument. The final state of the reservoir is also unique, depending only on the amount of heat ΔQ supplied to it, and independent of the speed at which that heat was supplied to it.

For process (a), we have

$$\Delta Q = C(T)\Delta T \tag{15}$$

where $C(T)$ is the equilibrium heat capacity of the system. For process (b), assume Joule heat ΔQ_J is generated. If we were to assume that the heat capacity of the system is still given by $C(T)$, it would be impossible for the system to reach the same final temperature $T - \Delta T$ upon transferring energy ΔQ to the reservoir, instead it would reach final temperature $T - \Delta T'$, with $\Delta T'$ satisfying

$$\Delta Q = C(T)\Delta T' + \Delta Q_J \tag{16a}$$

But, as argued in the above paragraph, the system reaches the same temperature $T - \Delta T$ in processes (a) and (b). Or alternatively, if the heat capacity doesn't change and the system in process (b) changes its temperature by the same amount ΔT as in process (a), it would have to transfer a different amount of heat to the reservoir

$$\Delta Q' = C(T)\Delta T + \Delta Q_J \tag{16b}$$

instead of ΔQ, again in contradiction with the assumptions.

So let us instead assume that the heat capacity of the system is different than the equilibrium one when the process occurs at a finite rate and involves Joule heat, let's call it $C_r(T) < C(T)$. We will then have for process (b)

$$\Delta Q = C_r(T)\Delta T + \Delta Q_J \tag{17}$$

transferred from the system to the reservoir, the same as in process (a), with the same change in temperature of the system. So under this assumption the first law of thermodynamics is not violated, energy is conserved. However, let's consider the change in entropy of the universe. In process (a) it is

$$\Delta S^{(a)}_{univ} = \frac{\Delta Q}{T_2} - \frac{\Delta Q}{T} + O((\Delta T)^2). \tag{18}$$

In process (b), Joule heat ΔQ_J is generated. Quantitatively, we obtain from Eqs. (4), (7) and (8)

$$\frac{\partial W}{\partial t} = \sigma_n (\frac{\partial \lambda_L}{\partial t})^2 \frac{H_0^2}{c^2} \frac{\pi h R \lambda_L(t)}{2} \tag{19}$$

and $\Delta Q_J = \int (\partial W/\partial t) dt$ is given, using that

$$\frac{\partial \lambda_L}{\partial t} = \frac{\partial \lambda_L}{\partial T}\frac{\partial T}{\partial t} \tag{20}$$

by

$$\Delta Q_J = \Delta T \frac{\partial T}{\partial t} \frac{\lambda_L(T)}{2R} \sigma_n (\frac{\partial \lambda_L}{\partial T})^2 \frac{H_0^2}{c^2} V \tag{21}$$

with

$$\frac{\partial T}{\partial t} = \frac{\kappa A}{C(T)} \frac{T - T_2}{d} \tag{22}$$

and $V = \pi R^2 h$ is the volume of the cylinder, d the thickness of the wall connecting the reservoir and system and A its area, and σ_n as given by Eq. (6) with $n_n(T)$.

Therefore, in process (b) entropy increases for two reasons. First, generation of Joule heat generates entropy:

$$\Delta S_J = \frac{\Delta Q_J}{T} + O((\Delta T)^2). \tag{23}$$

Note that ΔQ_J and hence ΔS_J is $O(\Delta T)$ and not $O((\Delta T)^2)$. Second, the transfer of the heat ΔQ from the system to the reservoir generates at least as much entropy as given by eq. (18), which is also $O(\Delta T)$. The reason we say 'at least' is because the transfer of heat

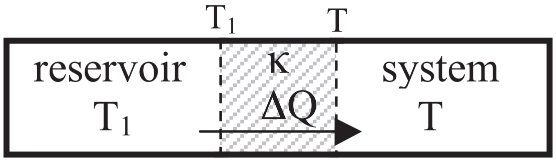

FIG. 4: Heating: the reservoir is at temperature T_1, the system is at temperature $T < T_1$. An amount of heat ΔQ flows from the reservoir to the system in a time t_0 that is inversely proportional to κ and to $(T_1 - T)$.

ΔQ out of the system lowers its entropy by $-\Delta Q/T$ *or less*, by Clausius inequality, if the process takes a finite time. Therefore, the change in entropy of the universe in process (b) is

$$\Delta S_{univ}^{(b)} \geq \Delta S_{univ}^{(a)} + \Delta S_J > \Delta S_{univ}^{(a)}. \tag{24}$$

However, entropy is a function of state. Therefore, the second law of thermodynamics is violated by Eq. (24).

We can also consider the reverse process, heating the superconductor below T_c, as shown in Fig. 4, where the inconsistency may be even clearer. With the system at temperature $T < T_1$, with T_1 the temperature of the reservoir, an amount of heat ΔQ will flow from the reservoir to the system and raise the temperature of the system by $\Delta T = \Delta Q/C(T)$. The change in entropy of the universe in this process if it occurs infinitely slowly (i.e. if $\kappa \to 0$) is

$$\Delta S_{univ} = -\frac{\Delta Q}{T_1} + \frac{\Delta Q}{T} + O((\Delta T)^2). \tag{25}$$

The time the process actually takes for finite κ is finite, inversely proportional to κ and to $T_1 - T$. The Joule heat generated ΔQ_J is given by Eq. (21) with $(T_1 - T)$ replacing $(T - T_2)$ in Eq. (22). We have to assume that through an unspecified process, which itself may violate the second law, a part of the absorbed ΔQ, in an amount that depends on κ, is used up in providing the work that propels the normal current, that decays by generation of ΔQ_J. In any event, Joule entropy

$$\Delta S_{extra} = \frac{\Delta Q_J}{T} + O((\Delta T)^2) \tag{26}$$

will be generated by the decay of the normal current that is added to the entropy Eq. (25), violating the second law.

VI. IS THERE A POSSIBLE RESOLUTION WITHIN THE CONVENTIONAL THEORY?

It has been suggested that one crucial flaw (1) in my argument may be the implicit assumption that the sample at intermediate times can be characterized by a uniform temperature T [5]. Another crucial flaw (2) may be that even assigning a well-defined temperature to the region where normal current is flowing may not be possible [5]. Another crucial flaw (3) may be that the relaxation time may be a function of momentum in the superconducting state rendering Eq. (5) invalid [6]. Another crucial flaw (4) may be that assuming that the only dissipative mechanism in the problem is the Joule heating may render my conclusion invalid [6]. Another crucial flaw (5) may be that within the conventional theory of superconductivity no electric field can exist in the superconductor unless the current exceeds a critical value. Another crucial flaw (6) may be that the final state of the reservoir is not unique because the reservoir is an infinite system. In the following I address those suggestions.

Regarding (3) and (4), I argue that even if those suggestions are valid they would not invalidate my argument. If there is another dissipative mechanism in the problem besides Joule heating (and in fact I believe there is within the conventional theory [7]) it would only make the inconsistency worse, since all I need is that there is some dissipation for the inconsistency to arise. Regarding Eq. (5), even if it needs to be replaced by a more complicated expression that would take into account a relaxation time that is a function of momentum and/or a non-local generalization of Ohm's law, it would not change the fact that it gives rise to dissipation. Furthermore, as discussed by Tinkham [1], Sect. 2.5, a two-fluid approximation with a normal conductivity given by Eq. (5) "is the standard working approximation for understanding electrical losses in superconductors" in situations with ac currents or applied electromagnetic fields, and there is no reason to expect within the conventional theory that the same would not apply to the situation considered here.

Let us consider the objection (1), that the sample may not be at a uniform temperature. First, it should be realized that the speed at which temperature equilibrates depends on another variable not included in the argument, namely the thermal conductivity of the sample, that is at our disposal. We may simply assume we have a sample with sufficiently high thermal conductivity that it homogenizes the temperature on a timescale much shorter than all other timescales in the problem.

Still, let us assume that for some unknown reason this does not happen. Considering the heating process of Fig. 4, let us assume the surface layer heats up and becomes hotter than the bulk, and becomes a 'subsystem' at temperature $T_h = T + \delta T$. One might argue that part of the incoming heat ΔQ provides energy to drive the Joule current in this subsystem at temperature T_h, and the resulting Joule heat is dumped into the bulk at the lower temperature T as in a 'heat engine', thus not violating the second law. The heat coming into the subsystem at temperature T_h would raise its entropy less than if its temperature was T, and this difference may account for the extra Joule entropy generated.

To counter this argument, we may simply assume that the sample is not heated from the surface but from the interior. Assume the sample is a hollow cylinder, with no magnetic field in the interior nor in the hollow cavity, so that supercurrent only flows near the outer surface as before. The heat ΔQ is added through the inner surface, so there is no mechanism for the outer surface layer to heat up beyond the bulk before generating Joule heat. As the heat ΔQ is coming in, the London penetration depth will increase, with a corresponding change in magnetic flux and associated Faraday field generated, and the generated Joule heat ΔQ_J will generate Joule entropy $\Delta Q_J/T$ thus violating the second law.

Regarding the suggestion (2) that assigning a well-defined temperature to the region where normal current is flowing may not be possible, we argue that even if so it does not eliminate the inconsistency. Particularly in the scenario described in the preceding paragraph. Furthermore one has to keep in mind that the thickness of the region where the current flows could even be a significant fraction of the volume of the system, at temperatures sufficiently close to T_c and with sufficiently small magnetic fields, and if the thermal conductivity of the sample is large and the process not very fast there is no reason to assume that a large portion of the system would have an undefined temperature.

Regarding the suggestion (5) that within the conventional theory of superconductivity no electric field can exist in a superconductor that is smaller than a critical value because it is shorted by the condensate, the suggestion is simply wrong, and reveals a deep lack of understanding of the conventional theory of superconductivity. Quoting from Tinkham chapter 2 [1], *"electric field also acts on the so-called "normal" electrons (really thermal excitations from the superconducting ground state, as we shall see in Chap. 3), which scatter from impurities, and can be described by Ohm's law."*

Regarding the suggestion (6) that the final state of the reservoir is not unique because the reservoir is an infinite system, the suggestion is also wrong and reveals a deep lack of understanding of thermodynamics. It is easy to see that in the scenario discuss in this paper the final state of the reservoir is unique whether the reservoir if infinite or finite. The system starts at temperature T_1, the (finite) "reservoir" starts at temperature $T_2 < T_1$, when they have reached thermal equilibrium they will both attain temperature T_3, with $T_2 < T_3 < T_1$. If the "reservoir is large, T_3 will be very close to T_2, if not it will not, but it doesn't matter. The key point is that the value of T_3 cannot depend on whether Joule heat was generated or not, by conservation of energy. To prove this we just have to remember that energy is a function of state. The energy of the system at temperatures T_1 and T_3 are fixed, so are the energies of the "reservoir" at temperatures T_2 and T_3. The "system plus "reservoir"" is the universe, there is nothing else. So by conservation of energy

$$E_{sys}(T_1) + E_{res}(T_2) = E_{sys}(T_3) + E_{res}(T_3) \tag{27}$$

If, by having the process go at different speed, with different Joule heat generated, the system plus reservoir would attain an equilibrium temperature T_4, we would have by conservation of energy

$$E_{sys}(T_1) + E_{res}(T_2) = E_{sys}(T_4) + E_{res}(T_4) \tag{28}$$

Therefore combining (27) and (28),

$$E_{sys}(T_3) + E_{res}(T_3) = E_{sys}(T_4) + E_{res}(T_4) \tag{29}$$

hence from (29)

$$E_{sys}(T_3) - E_{sys}(T_4) = E_{res}(T_4) - E_{res}(T_3) \tag{30}$$

If T_3 is not identical to T_4, this equation ((30)) implies that either the system or the reservoir have a negative heat capacity. I.e. for example, if $T_3 > T_4$ and the left side of (30) is positive, the right side is positive hence $E_{res}(T_3) - E_{res}(T_4)$ is negative, hence the 'reservoir' has negative heat capacity. But thermodynamic systems with negative heat capacity can't exist. Therefore, $T_3 = T_4$. Therefore, the system plus the (finite) 'reservoir' have to reach a unique final equilibrium temperature, independent of how much Joule heat is generated in the process. Therefore, all the arguments in this paper apply to the system plus reservoir reaching a unique final temperature T_3 with $T_2 < T_3 < T_1$.

Referees of this paper and ref. [6] have also raised the question whether an inconsistency as pointed out here would also arise in magnetic systems, in particular in ferromagnets. When the temperature is changed and the magnetization changes, a normal current would be generated by Faraday's law and dissipation would occur that may also be expected to depend on the speed of the cooling process.

I have not studied these issues for the case of magnetic systems, so I don't know for sure but strongly suspect that the same problem doesn't arise there. There are important differences between ferromagnetism and superconductivity. (i) There is hysteresis in ferromagnets, there is none in type I superconductors; (ii) The ferromagnetic transition is 2nd order, the superconducting one (type I in a magnetic field) is first order; (iii) The Clausius-Clapeyron equation applies to superconductors, not to ferromagnets. So the superconducting transition is more analogous to the water-ice or water-vapor transition than to the ferromagnetic transition; (iv) in magnetic systems there are necessarily other dissipation mechanisms at play: if a magnetic field is applied to a magnetic moment not parallel to the field, the magnetic moment will precess around the magnetic field direction, and its component along the field will not change, hence the magnetization will not change, unless there is a damping mechanism, which will generate entropy. So it is possible that the Joule heating from the Faraday field and this other dissipative mechanism combine to give a total change in entropy that depends only on initial and final states and not on the speed of the process.

In this connection I would also like to quote from Fritz London's 1950 book on superconductivity [8] (emphasis added): *"So far all attempts to develop a molecular theory of superconductivity have taken it more or less for granted that it is necessary to assume a great number of different equilibrium states corresponding to different spontaneous currents differing in direction and in strength. Apparently this idea originated from a* ***quite unwarranted analogy with ferromagnetism. In the case of ferromagnetism it is in fact possible to construct a separate state for every orientation of the spontaneous magnetic moment. However, for superconductivity such a construction is of no avail.*** *If the various supercurrents really were to correspond to a continuum of different quantum states it would seem extremely hard to understand how a supercurrent could resist so many temptations to dissipate into the other states. Moreover, in this case,* ***we should expect to find hysteresis whenever a change of direction or of strength of a supercurrent is produced, say by changing the direction or the strength of an applied magnetic field - something similar to ferromagnetic hysteresis. Nothing of this kind has ever been observed with superconductors,*** *and indeed any hysteresis would be quite incompatible with all evidence we have concerning the peculiar mobility by which the supercurrents must adjust themselves to the slightest changes of an applied external field in order to maintain $B = 0$... The electrodynamics of the superconductor leads to an entirely different concept. In an isolated, simply connected superconductor and for a given applied magnetic field, there is just one stable current distribution, the direction and strength of the current everywhere being determined by the direction and strength of the external field ...We also saw that for a given external field,* ***in contrast to ferromagnetism, an isolated, simply or multiply connected superconductor can be assumed to have only one true equilibrium state".***

Finally, in connection with the many comments I have received from referees pointing to the many successes of the conventional theory of superconductivity, I would simply like to point out that it is sufficient that there is one situation where the inconsistency pointed out in this paper clearly exists to validate our argument. Achilles' heel doesn't have to be more than a tiny fraction of the entire body area.

VII. DISCUSSION

The Faraday electric field is a consequence of Maxwell's equations and is unavoidable. The fact that electric fields in superconductors give rise to dissipation is well known from experiments with ac currents or electromagnetic waves incident on superconductors [9], and is predicted by BCS theory [9]. So how can this inconsistency be resolved?

If the processes occurs always infinitely slowly, the Joule heat and associated entropy go to zero and the inconsistency is resolved. However, there is no mechanism to make these processes proceed infinitely slowly if κ is large. Furthermore, we know from experiments that superconductors in magnetic fields can be cooled or heated and reach equilibrium states at the new temperatures in finite time.

Another way to resolve this inconsistency would be to assume that the final state depends on the process. Neither I nor (I suspect) anybody else is willing to go back to that notion, that was discarded in 1933. The contrary notion is an integral part of the conventional theory.

I argue that the only other way to resolve this inconsistency is to assume that in the particular situation considered here, where the electric field arises from a change in temperature, the superconductor behaves differently than in other situations with electric fields, namely here no normal current is generated and no dissipation takes place.

That is not predicted by the conventional theory [1]. In addition, within the conventional theory that is impossible, for the following reason. From Ampere's law,

$$\oint \vec{B} \cdot d\vec{\ell} = \frac{4\pi}{c} I \tag{31}$$

where I is the total current, yielding

$$I = \frac{c}{4\pi} h H_0. \tag{32}$$

where h is the height of the cylinder. Therefore, the total current I is independent of temperature. However, the Faraday electric field transfers momentum to the supercurrent, as well as to the body as a whole. In order for the current to stay the same, there has to be a mechanism for momentum transfer between electrons and the body as a whole.

Within the conventional theory of superconductivity, the only way to transfer momentum between electrons and ions is through scattering processes involving normal electrons, the same processes that give rise to normal resistivity and Joule heat in the normal state [10]. If these processes occur at a finite rate as in the situation considered here, finite Joule heat and Joule entropy will necessarily be generated. Therefore, the inconsistency cannot be resolved within the conventional theory.

The only way to transfer momentum between electrons and ions without dissipation other than infinitely slowly is if electrons have negative effective mass. If so, an external force acting on the electron gives rise to acceleration in opposite direction to the force because the difference in momentum is transferred to the body, without scattering processes and associated dissipation.

This then implies that to resolve the inconsistency pointed out in this paper charge carriers in superconductors have to be holes [11] rather than electrons. This is not required within the conventional theory but is required within the alternative theory of hole superconductivity [12]. We have shown that within that theory there is momentum transfer between electrons and ions without dissipation in the normal-superconductor and superconductor-normal transitions in the presence of a magnetic field [7, 13, 14].

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[1] M. Tinkham, "Introduction to superconductivity", McGraw Hill, New York, 1996.
[2] M. von Laue, "Theory of Superconductivity", Academic Press, New York, 1952.
[3] F. Reif, 'Fundamentals of statistical and thermal physics, McGraw-Hill, New York, 1965.
[4] J. E. Hirsch, arXiv:1907.11273 (2019).
[5] B. I. Halperin, private communication (2019).
[6] A. J. Leggett, private communication (2019).
[7] J.E. Hirsch, "Entropy generation and momentum transfer in the superconductor to normal phase transformation and the consistency of the conventional theory of superconductivity", Int. J. Mod. Phys. B, Vol. 32, 1850158 (2018).
[8] F. London, "Superfluids", Vol. I, Dover, New York, 1961, p. 142.
[9] Reference [1], Sects. 2.5, 3.10.
[10] B. I. Halperin, private communication (2017).
[11] N. W. Ashcroft and N. D. Mermin, "Solid State Physics", Chapter 12, Saunders College Publishing, Fort Worth, 1976.
[12] See references in http://physics.ucsd.edu/∼jorge/hole.html.
[13] J.E. Hirsch, "Momentum of superconducting electrons and the explanation of the Meissner effect", Phys. Rev. B **95**, 014503 (2017).
[14] J. E. Hirsch, 'On the reversibitity of the Meissner effect and the angular momentum puzzle', Annals of Physics **373**, 230 (2016).