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How Alfven's theorem explains the Meissner effect

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Alfven's theorem states that in a perfectly conducting fluid magnetic field lines move with the fluid without dissipation. When a metal becomes superconducting in the presence of a magnetic field, magnetic field lines move from the interior to the surface (Meissner effect) in a reversible way. This indicates that a perfectly conducting fluid is flowing outward. I point this out and show that this fluid carries neither charge nor mass, but carries *effective mass*. This implies that the effective mass of carriers is lowered when a system goes from the normal to the superconducting state, which agrees with the prediction of the unconventional theory of hole superconductivity and with optical experiments in some superconducting materials. The 60-year old conventional understanding of the Meissner effect ignores Alfven's theorem and for that reason I argue that it does not provide a valid understanding of real superconductors.

Keywords: Alfven's theorem; Meissner effect; frozen field lines.

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1. Introduction

When a conducting fluid moves, magnetic field lines tend to move with the fluid, as a consequence of Faraday's law.[1] If the fluid is perfectly conducting, the lines are "frozen" in the fluid. That is known as "Alfven's theorem".[2] No dissipation occurs when a perfectly conducting fluid together with magnetic field lines move. If the fluid is not perfectly conducting, there will be relative motion of magnetic field lines with respect to the fluid and Joule heat will be dissipated.[1] Even for non-perfectly conducting fluids, as P. H. Roberts points out,[3] "*Alfven's theorem is also helpful in attacking the problem of inferring unobservable fluid motions from observed magnetic field behavior*". For example, measurements of magnetic field variations near one of Jupiter's moons demonstrated the existence of an unobservable conducting fluid below its surface.[4] This paper is based on Roberts' principle.

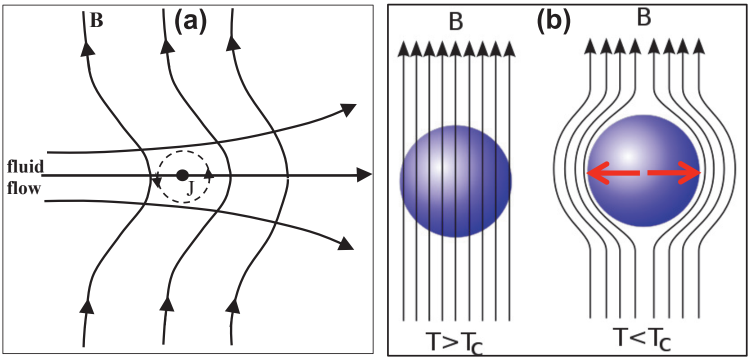

Fig. 1. (a) An example illustrating Alfven's theorem for a conducting fluid.[1] Fluid flow across magnetic field lines causes the field lines to bow out. (b) The Meissner effect in a superconductor. The red arrows show the hypothesized motion of fluid, by analogy to the left panel.

In the transition from normal metal to superconductor in the presence of a magnetic field, magnetic field lines move out of the interior of the system. This is called the Meissner effect. The transition is thermodynamically reversible, i.e. it occurs without dissipation under ideal conditions. In both the normal and the superconducting states of the metal there are mobile electric charges, which certainly qualify as a conducting fluid. Thus it is logical to infer that the motion of magnetic field lines in the normal-superconductor transition is associated with the motion of charges, specifically that the motion of magnetic field lines *reflects* the motion of charges. In this paper we propose that this is indeed the case, and explain what the nature of this conducting fluid is and *what this fluid motion carries with it* in addition to the magnetic field. The relation between Alfven's theorem and the Meissner effect is shown schematically in Fig. 1.

Instead, the conventional (BCS) theory of superconductivity[5] says that the outward motion of magnetic field lines in the normal-superconductor transition is determined by quantum mechanics and energetics and is *not* associated with the outward motion of any charges. We will argue that this is incorrect.

In earlier work we have used related concepts to explain the physics of the Meissner effect based on the theory of hole superconductivity proposed to describe all superconducting materials.[6] This will be discussed later in the paper.

2. Why Conventional BCS Theory Does not Explain the Meissner Effect

Because in this paper we propose an understanding of the Meissner effect which is radically different from the generally accepted one, we first briefly review here the reasons for why we argue that the conventional theory of superconductivity does not explain the Meissner effect.[6]

In addressing the Meissner effect, Bardeen, Cooper and Schrieffer (BCS)[7] considered the linear response of a system *in the BCS state* to the perturbation

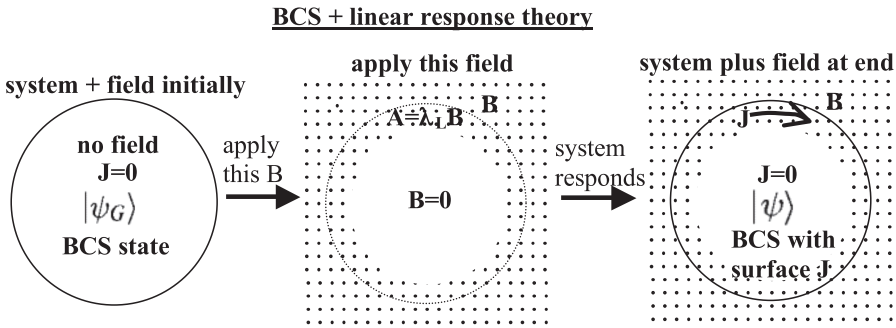

Fig. 2. The BCS explanation of the Meissner effect. The system (cylinder, top view) is initially in the BCS state (left panel) with no magnetic field. Its linear response to the magnetic field shown in the middle panel (dots) is computed to first-order in the magnetic field. The result is the state shown in the right panel, with a surface current J circulating.

created by a magnetic field, as shown in Fig. 2. The perturbing Hamiltonian is the linear term in the magnetic vector potential $\vec{A}$ that results from the kinetic energy $(\vec{p} - (e/c)\vec{A})^2/2m$:

$$H_1 = \frac{ie\hbar}{2mc}\sum_i(\vec{\nabla}_i \cdot A + \vec{A} \cdot \vec{\nabla})\,. \tag{1}$$

This perturbation causes the BCS wavefunction $|\Psi_G\rangle$ to become, to first-order in $\vec{A}$

$$|\Psi\rangle = |\Psi_G\rangle - \sum_n \frac{\langle\Psi_n|H_1|\Psi_G\rangle}{E_n}|\Psi_n\rangle\,, \tag{2}$$

where $|\Psi_n\rangle$ are states obtained from the BCS state $|\Psi_G\rangle$ by exciting 2 quasiparticles, and E_n is the excitation energy. The expectation value of the current operator $\vec{J}_{op}$ with this wavefunction gives the electric current $\vec{J}$:

$$\vec{J} = \langle\Psi|\vec{J}_{op}|\Psi\rangle = -\frac{c}{4\pi}K\vec{A} \tag{3a}$$

where K is the "London Kernel".[5] We omit wavevector dependence here for simplicity. In the long wavelength limit this calculation yields[7]

$$K = \frac{1}{\lambda_L^2}\,, \tag{3b}$$

where λ_L is the London penetration depth. Equation (3) is the (second) London equation. In combination with Ampere's law, Eq. (3) predicts that the magnetic field does not penetrate the superconductor beyond a distance λ_L from the surface, where the current $\vec{J}$ circulates, as shown schematically in Fig. 2 (right panel).

This calculation is in essence what BCS and others following BCS did[8–15] in examining issues associated with gauge invariance. Note that it *uses only the BCS wavefunction in and around the BCS state*, namely the ground state wavefunction

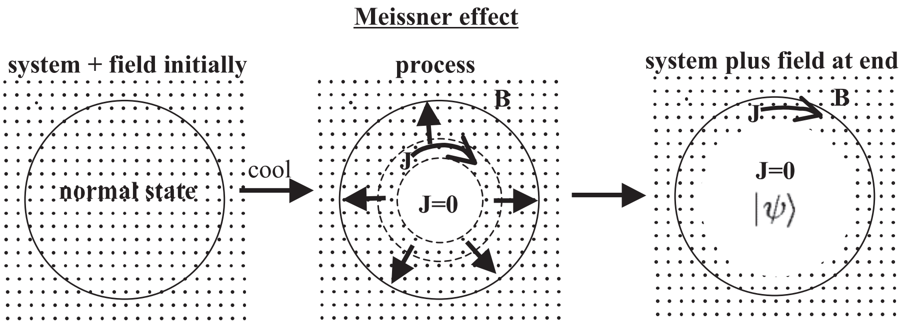

Fig. 3. What the Meissner effect really is: the *process* by which a normal metal becomes superconducting in the presence of a magnetic field throughout its interior initially. The simplest route in this process (not the only one) is depicted in this figure. The superconducting region (white region) expands gradually from the center to fill the entire volume, expelling the magnetic field in the process.

$|\Psi_G\rangle$ and the wavefunctions $|\Psi_n\rangle$ that result from breaking one Cooper pair at a time. The wavefunction of the normal metal never appears in these calculations.

We argue that that this does *not* explain the Meissner effect. The Meissner effect is what is shown in Fig. 3: the *process* by which a system starting in the normal metallic state expels a magnetic field in the process of becoming a superconductor. It cannot be explained by starting from the assumption that the system is in the final BCS state and gets perturbed by H_1. Explaining this process requires explaining how the interface between normal and superconducting regions moves (center panel in Fig. 3). That is the subject of this paper. Because calculations of the sort described in Eqs. (1)–(3) contain no information about what is the nature of the initial state when the Meissner effect starts, namely the normal metal, they cannot be a microscopic derivation of the Meissner effect.

There have also been calculations[16–18] of the kinetics of the transition process using time-dependent Ginzburg–Landau theory.[19–21] However that formalism is phenomenological and involves a first-order differential equation in time with *real* coefficients for the time evolution of the order parameter. Hence it describes *irreversible* time evolution, and is therefore not relevant to the Meissner effect for type-I superconductors, which is a *reversible* process.

During the process of field expulsion, as well as its reverse, the process where a superconductor with a magnetic field excluded turns normal and the field penetrates, a Faraday electric field is generated that opposes the process. This electric field drives current in direction opposite to the current that develops. So it is necessary to explain:

(i) How can a Meissner current start to flow in direction opposite to the Faraday electric force resisting magnetic flux change (Lenz's law)?
(ii) How is the angular momentum of the developing supercurrent compensated so that momentum conservation is not violated?

(iii) When a supercurrent stops, what happens to the angular momentum that the supercurrent had?
(iv) How can a supercurrent stop without generation of Joule heat and associated with it an irreversible increase in the entropy of the universe that is known not to occur?

None of these questions are addressed in the BCS literature.

3. Alfven's Theorem

When a conducting fluid moves with velocity $\vec{u}$ in the presence of electric and magnetic fields $\vec{E}$ and $\vec{B}$, electromagnetism dictates that an electric current density[1]

$$\vec{J} = \sigma\left(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}\right) \tag{4}$$

exists, where σ is the electrical conductivity of the fluid. In particular, for a perfectly conducting fluid $\sigma = \infty$ and

$$\vec{E} = -\frac{1}{c}\vec{u} \times \vec{B}\,. \tag{5}$$

Figure 1 shows in the left panel qualitatively how this leads to Alfven's theorem. The horizontal motion of the fluid generates a current J pointing out of the paper which generates a counterclockwise magnetic field indicated by the dashed circle, which added to the original magnetic field gives curvature to the magnetic field lines that were originally straight. Thus, the magnetic field lines bend in the direction of the fluid motion. Analogously we suggest in this paper that the motion of magnetic field lines in the right panel of Fig. 1 is associated with motion of a conducting fluid as indicated by the red arrows.

Using Faraday's and Ampere's laws,

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,, \tag{6}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} \tag{7}$$

Eq. (4) yields

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \frac{c^2}{4\pi\sigma}\nabla^2\vec{B} \tag{8}$$

and in particular for a perfectly conducting fluid

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B})\,. \tag{9}$$

Equation (9) implies that magnetic field lines are frozen into the fluid. The proof is given in Appendix A. This implies that for a perfectly conducting fluid outward motion of field lines is *necessarily* associated with outward motion of the fluid.

For generality, we could assume that in addition to the current given by Eq. (4) there is a "quantum supercurrent" $\vec{J}_s$ generated by an unknown quantum mechanism provided by BCS or another microscopic theory:

$$\vec{J} = \sigma\left(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}\right) + \vec{J}_s \,. \tag{10}$$

Instead of Eq. (8), we would obtain from Eq. (10)

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \frac{c^2}{4\pi\sigma}\nabla^2 \vec{B} + \frac{c}{\sigma}\vec{\nabla} \times \vec{J}_s \,. \tag{11}$$

Consider a long metallic cylinder initially in the normal state with uniform magnetic field in the $\hat{z}$ direction. In cylindrical coordinates and assuming translational invariance in the $\hat{z}$ and $\hat{\theta}$ (azimuthal) directions Eq. (11) yields for the time evolution of the magnetic field $\vec{B} = B(r,t)\hat{z}$

$$\frac{\partial B(r,t)}{\partial t} = -\frac{1}{r}\frac{\partial(ru_r B)}{\partial r} + \frac{c}{\sigma}\frac{1}{r}\frac{\partial}{\partial r}(rJ_{s\theta}) \,. \tag{12}$$

Note that the last term, the contribution of the "quantum supercurrent" to the time evolution of the magnetic field, decreases as σ increases. Thus it is natural to conclude that for large σ at least the time evolution of the magnetic field is dominated by the first term in Eq. (12), which requires *radial* motion of the fluid, $u_r \neq 0$, i.e. *motion of the conducting fluid in direction perpendicular to the field lines.*

Within the conventional theory of superconductivity[5] $u_r = 0$ and the expulsion of magnetic field has to be explained solely by the last term in Eq. (12). The explanation has to be valid for any value of σ, since normal metals of widely varying conductivities expel magnetic fields when they become superconducting. How this happens within the conventional theory has not been explained in literature.

Instead, in this paper we will assume that the last term in Eq. (12) does not exist and explain the Meissner effect in a natural way through the outward motion of a perfectly conducting fluid.

4. The Puzzle

A perfectly conducting fluid moving from the interior to the surface when a normal metal becomes superconducting would satisfy Eq. (9) and as a consequence, as shown in Appendix A, would carry the magnetic field lines with it and explain the Meissner effect. However, there are obvious problems with this explanation:

(1) If the fluid is charged, this motion would result in an inhomogeneous charge distribution, costing an enormous electrostatic potential energy. So this cannot happen.

(2) Even if the fluid is charge neutral, like a neutral plasma composed of electrons and ions with equal and opposite charge densities, outward motion would be associated with outward mass flow, generating an enormous mass imbalance. This cannot happen. Plasmas cannot expel magnetic fields by outward motion.

(3) Furthermore, in a solid the positive ions cannot move a finite distance. The only mobile charges are electrons.

So in order to explain the Meissner effect using Alfven's theorem we need to identify a charge-neutral mass-neutral electricity-conducting fluid that moves from the interior to the surface in the process of the metal becoming superconducting without dissipation.

And this poses an additional question: if this fluid carries neither charge nor mass, what does it carry?

The next section provides the answers.

5. The Answers

Charge carriers in electronic energy bands can be electrons or holes.[22] We will need both to explain how magnetic flux is expelled.

Consider a long metallic cylinder of radius R, of a material that is a type-I superconductor, in a uniform applied magnetic field $H = H_c(T)$ parallel to its axis, where $H_c(T)$ is the critical magnetic field at temperature T,[5] that is initially at temperature higher than T. When the system is cooled to temperature T it will become superconducting and expel the magnetic field to a surface layer of thickness λ_L, the London penetration depth at that temperature, typically hundreds of $\mathring{A}$, given by[5]

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m^* c^2} \tag{13}$$

with n_s the density of superfluid carriers and m^* their effective mass.

Assume that the transition proceeds as follows. Initially, in a central core of radius r_c a perfectly conducting fluid of n_s electrons and n_s holes per unit volume forms, both carriers with effective mass m^*, with r_c given by

$$r_c = \sqrt{2R\lambda_L} \tag{14}$$

as shown in the left panel of Fig. 4. Then, assume this fluid flows radially outward until it reaches the surface. Assuming it is incompressible, it will at the end occupy an annulus of thickness λ_L adjacent to the surface, since

$$\pi r_c^2 = 2\pi R\lambda_L\,. \tag{15}$$

Because of Alfven's theorem, the magnetic field lines that were initially in the region $r \leq r_c$ flow out with the fluid. No magnetic field line can cross either the inner or the outer boundary of this fluid, therefore the magnetic field lines that were outside initially are pushed further out as the fluid moves out, and in the interior no magnetic field ever exists. The end result is what is shown on the right panel of Fig. 4: no magnetic field in the region $r < R - \lambda_L$. The magnetic field has been expelled from the interior and remains only in a surface layer of thickness λ_L, as occurs in the Meissner effect.

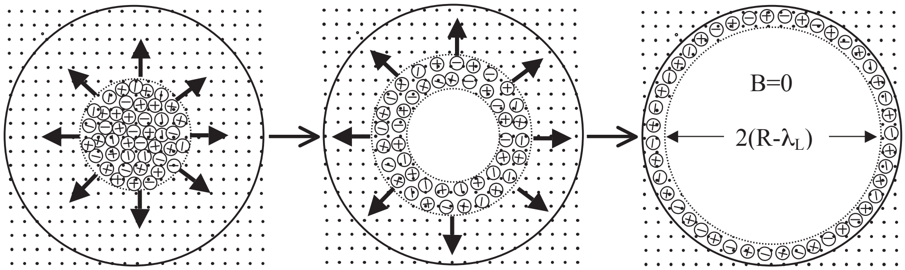

Fig. 4. Simple model for the Meissner effect in a cylinder (top view). A perfectly conducting fluid of electrons and holes occupies initially the central region (of radius r_c) of a cylinder of radius R (left panel) and flows to the surface where it occupies a ring of thickness λ_L. Points indicate magnetic field pointing out of the paper, initially uniformly distributed across the cylinder cross section.

There is however a small difference. Frozen field lines would imply that in the final state the magnetic field is uniform in the region $R - \lambda_L < r < R$ and drops discontinuously to zero at $r = R - \lambda_L$. This is not so in the Meissner effect, rather the magnetic field near the surface is given by (to the lowest order in λ_L/R)

$$H(r) = H_c e^{(r-R)/\lambda_L} . \tag{16}$$

The reason for the difference is that in assuming that Eq. (9) is valid at all times we are ignoring transient effects and the inertia of charge carriers. This is a minor difference, in particular the magnetic field flux through the region $r \leq R$ is the same for Eq. (16) as it is for a uniform H_c between $R - \lambda_L$ and R.

The fluid that flowed out is charge neutral, by assumption, so no charge imbalance results from this process. Furthermore *no mass imbalance* results from this process either. To understand this one has to remember that "holes" are not real particles, they are a theoretical construct.[22] When holes flow in a given direction, physical mass is flowing in the opposite direction. This is illustrated in Fig. 5. So the process that we envision shown in Fig. 4 would result if we have conduction in two bands, one close to empty and the other one close to full, with the same density of electrons and holes. This is depicted in Fig. 6.

But if neither charge nor mass flowed out, what quantity is being transported out in the process shown in Fig. 4?

The answer is, *effective mass*. The effective mass *of electrons* is given by the curvature of the energy bands in Fig. 6. Having holes with positive charge and positive effective mass flowing out is equivalent to having electrons with negative charge and negative effective mass flowing in, as shown in Fig. 5. So the electron band carriers carry *out* positive effective mass, and the hole band carriers carry *in* negative effective mass, which is equivalent to also carrying out positive effective mass. This implies that there is a net *outflow of effective mass* in the process where a metal turning superconductive expels magnetic field. In addition to expelling magnetic field, the system expels effective mass

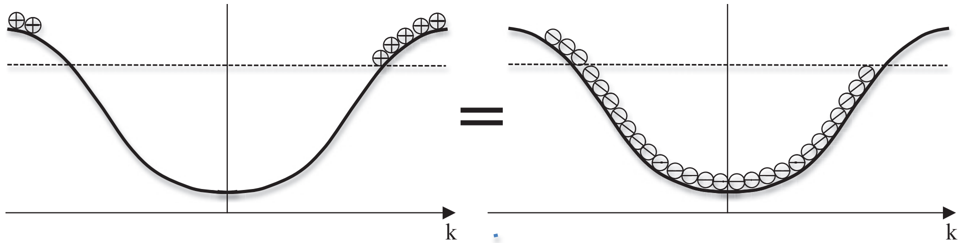

Fig. 5. Holes flowing in the positive k direction (left panel) corresponds to electrons flowing in the negative k direction (right panel).

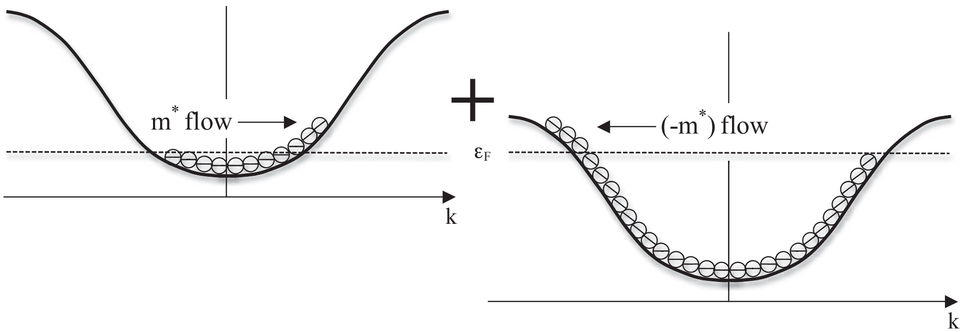

Fig. 6. The left panel shows electrons flowing out (positive k direction) carrying *out* positive effective mass. The right panel shows holes flowing out (see left panel of Fig. 5), carrying *in* (negative k direction) *negative* effective mass, which is equivalent to carrying out positive effective mass also. Both of these flows occur in Fig. 4. ϵ_F is the Fermi energy.

As a consequence, in the process of a metal turning superconducting the effective mass of the carriers in the system decreases. We discuss this further in Sec. 8.

6. Kinetics of the Fluid Motion

Equation (9) guarantees that no magnetic field lines can cross the boundaries of our annulus of perfectly conducting fluid as it moves outward, as shown in Appendix A. Let us consider the current distribution. The details of the current distribution will depend on the initial conditions. We assume initial conditions so that it is only the electrons that have azimuthal velocity. Figure 7 shows an intermediate state in the process.

$r_0(t)$ is the inner radius of the annulus of fluid that is moving outward with speed $\dot{r}_0$. The fluid velocity field is given by

$$\vec{u}(r) = \dot{r}_0 \frac{r_0}{r} \hat{r} \,. \tag{17}$$

We assume that $\dot{r}_0$ is of order of the speed at which superconductors expel magnetic fields in experiments,[23] typically mm/s, hence *much* smaller than the speed of light. The magnetic field is zero for $r \leq r_0$ according to Alfven's theorem, and it is given

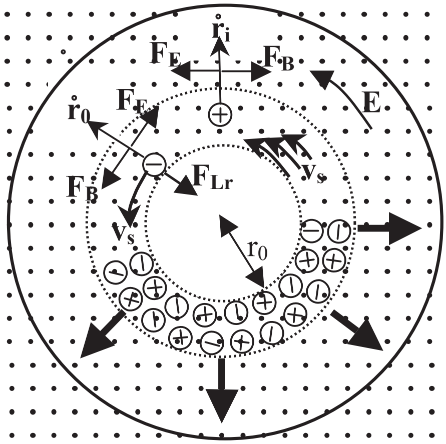

Fig. 7. Expulsion of magnetic field (dots) through motion of perfectly conducting fluid. The charge distribution has azimuthal symmetry but only some of the carriers are shown for clarity. Both electrons and holes move radially out with speed $\dot{r}_0$. In addition electrons have azimuthal speed v_s that nullifies the magnetic field in the interior. The electric and magnetic Lorentz forces are balanced in the azimuthal direction for both electrons and holes. For electrons there is also a radial Lorentz force F_{Lr} that is balanced by quantum pressure (see text).

by H_c for $r \gg r_0$. It cannot go to zero discontinuously at r_0 unless the current density at $r = r_0$ is infinite. So we assume it goes continuously to zero in a region of thickness λ adjacent to the surface where current flows. It is natural to assume that the decay is exponential, so we assume the form

$$\vec{H}(r) = H_c(1 - e^{(r_0 - r)/\lambda})\hat{z}\,. \tag{18}$$

In Sec. 7, we will show that the decay is indeed exponential and that $\lambda = \lambda_L$, with λ_L the London penetration depth given by Eq. (13).

Using Faraday's law and the fact that the magnetic field is zero in the deep interior we obtain for the Faraday electric field

$$\vec{E}(r) = \frac{\dot{r}_0}{c}\frac{r_0}{r}H_c(1 - e^{(r_0 - r)/\lambda})\hat{\theta} \tag{19}$$

(to lowest order in λ/r). The azimuthal velocity for the electrons in the annulus is

$$\vec{v}_s(r) = -\frac{c}{4\pi e n_s \lambda}H_c e^{(r_0 - r)/\lambda}\hat{\theta} \tag{20}$$

giving rise to azimuthal current density

$$\vec{J} = n_s e \vec{v}_s = -\frac{c}{4\pi\lambda}H_c e^{(r_0 - r)/\lambda}\hat{\theta}\,. \tag{21}$$

The current density Eq. (21) satisfies Ampere's law

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{J}. \tag{22}$$

From Eq. (18)

$$\frac{\partial \vec{H}}{\partial t} = -\frac{\dot{r}_0}{\lambda}\vec{H}\,. \tag{23}$$

and Eq. (9) is satisfied to lowest order in λ_L/r, with $\vec{u}$ given by Eq. (17). The electric field, magnetic field and fluid velocity equations, Eqs. (18), (16) and (17) respectively are related by the condition

$$\vec{E} = -\frac{1}{c}\vec{u}\times\vec{H} \tag{24}$$

in agreement with Eq. (5).

The Lorentz force

$$\vec{F}_L = q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right) \equiv \vec{F}_E + \vec{F}_B \tag{25}$$

in the azimuthal direction is zero for both electrons and holes with $v_r = u(r)$, as shown schematically in Fig. 7. For electrons there is also a Lorentz force in the radial direction

$$F_{Lr} = \frac{e}{c}v_s H\hat{r} = -\frac{1}{4\pi n_s\lambda}e^{2(r_0-r)/\lambda}H_c^2\hat{r}\,, \tag{26}$$

so in order for this fluid to move outward there has to be an outward force that compensates the inward force Eq. (26). That outward force $F_r = -F_{Lr}$ (per unit area) is called "Meissner pressure" and it arises from the difference in energy between normal and superconducting states. From Eq. (26) we obtain the work done by F_r per unit area per unit time:

$$\int_{r_0}^{\infty} dr F_r v_r n_s = \frac{H_c^2}{8\pi}\dot{r}_0 \tag{27}$$

which is the rate of change of magnetic energy per unit area as the phase boundary moves. This energy is provided by the condensation energy of the superconductor.

7. Dynamics of the Fluid Motion

Here we show that the magnetic and velocity fields discussed in Sec. 5 indeed have exponential dependence on r as assumed and decay length λ given by the London penetration depth λ_L [Eq. (13)].

The equation of motion for electrons of effective mass m^* in electric and magnetic fields in a perfectly conducting fluid is

$$\frac{d\vec{v}}{dt} = \frac{e}{m^*}\vec{E} + \frac{e}{m^*c}\vec{v}\times\vec{H}\,. \tag{28}$$

Using the relation between total and partial time derivatives, Eq. (28) becomes

$$\frac{\partial\vec{v}}{\partial t} + \vec{\nabla}\left(\frac{v^2}{2}\right) - \vec{v}\times(\vec{\nabla}\times\vec{v}) = \frac{e}{m^*}\vec{E} + \frac{e}{m^*c}\vec{v}\times\vec{H}\,. \tag{29}$$

In cylindrical coordinates, the velocity field is

$$\vec{v}(r,t) = \vec{v}_\theta(r,t)\hat{\theta} + \frac{r_0}{r}\dot{r}_0\hat{r}\,, \tag{30}$$

so for the azimuthal direction Eq. (29) yields

$$\frac{\partial v_\theta}{\partial t} + \dot{r}_0\frac{r_0}{r^2}\frac{\partial}{\partial r}(rv_\theta) = \frac{e}{m^*}E + \frac{e}{m^*c}\dot{r}_0\frac{r_0}{r}H\,. \tag{31}$$

On the other hand, by taking the curl on both sides of Eq. (29) we find

$$\frac{\partial \vec{w}}{\partial t} = \vec{\nabla} \times [\vec{v} \times \vec{w}] \tag{32}$$

with

$$\vec{w} \equiv \vec{\nabla} \times \vec{v} + \frac{e}{m^*c}\vec{H}\,, \tag{33a}$$

$$\vec{w} = w(r,t)\hat{r}\,, \tag{33b}$$

and from Eq. (33a)

$$w(r,t) = \frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) + \frac{e}{m^*c}H(r,t)\,. \tag{34}$$

In cylindrical coordinates Eq. (32) is

$$\frac{\partial w}{\partial t} = -\frac{r_0}{r}\dot{r}_0\frac{\partial w}{\partial r} \tag{35}$$

that is satisfied by

$$w(r,t) = g\left(r - \frac{r_0}{r}\dot{r}_0 t\right). \tag{36}$$

with an arbitrary function g. Now at $t = 0$ we have from Eq. (34)

$$w(r,t=0) = \frac{e}{m^*c}H_c\,, \tag{37}$$

for all r, since the fluid has not started to move. Therefore, from Eq. (36)

$$w(r,0) = g(r) = \frac{e}{m^*c}H_c\,, \tag{38}$$

for all r. Therefore, w is simply given by

$$w(r,t) = \frac{e}{m^*c}H_c \tag{39}$$

and from Eq. (34)

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) = \frac{e}{m^*c}(H_c - H(r,t))\,. \tag{40}$$

We now replace Eq. (40) in the equation of motion (31) and obtain

$$\frac{\partial v_\theta(r,t)}{\partial t} + \dot{r}_0\frac{r_0}{r}\frac{e}{m^*c}H_c = \frac{e}{m^*}E(r,t)\,. \tag{41}$$

Now from Ampere-Maxwell's law

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{42}$$

and using that

$$J_\theta = n_s e v_\theta \,, \tag{43}$$

Eq. (42) yields

$$\frac{\partial E}{\partial t} = -c\frac{\partial H}{\partial r} - 4\pi n_s e v_\theta \,. \tag{44}$$

Taking the time derivative of Eq. (41) and using Eq. (44)

$$\frac{\partial^2 v_\theta}{\partial t^2} = -\frac{ec}{m^*}\frac{\partial H}{\partial r} - \frac{4\pi n_s e^2}{m^*} v_\theta \,. \tag{45}$$

Taking the space derivative of Eq. (40)

$$\frac{\partial H}{\partial r} = -\frac{m^* c}{e}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) \tag{46}$$

and replacing Eq. (46) in Eq. (45)

$$\frac{1}{c^2}\frac{\partial^2 v_\theta}{\partial t^2} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) - \frac{4\pi n_s e^2}{m^* c^2} v_\theta \,. \tag{47}$$

Equation (47) describes the full time dependence of the process discussed in Sec. 6 including the initial transient when the fluid starts to move and the azimuthal current is established. The initial conditions are

$$v_\theta(r, t = 0) = 0 \,, \tag{48a}$$

$$\left.\frac{\partial v_\theta(r > 0, t)}{\partial t}\right|_{t=0} = 0 \,, \tag{48b}$$

$$\left.\frac{\partial v_\theta(r = 0, t)}{\partial t}\right|_{t=0} = \frac{e}{m^* c}\dot{r}_0 H_c \,. \tag{48c}$$

Now in Sec. 6, we assumed for the azimuthal velocity in the steady state situation

$$v_\theta(r, t) = -\frac{c}{4\pi e n_s \lambda} H_c e^{(\dot{r}_0 t - r)/\lambda} \,. \tag{49}$$

Its second time derivative is

$$\frac{1}{c^2}\frac{\partial^2 v_\theta}{\partial t^2} = \left(\frac{\dot{r}_0}{c}\right)^2 \frac{v_\theta}{\lambda^2} \,. \tag{50}$$

Since we assume $\dot{r}_0 \ll c$, we conclude that Eq. (50) is completely negligible and hence that after an initial transient where the velocity field is established, the left-hand side of Eq. (47) is completely negligible. In steady state then Eq. (47) yields

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) - \frac{1}{\lambda_L^2} v_\theta = 0 \tag{51}$$

with

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m^* c^2} \,. \tag{52}$$

the same as Eq. (13) for superconductors.

The exact solution of Eq. (51) is simply obtained in terms of Bessel functions.[24] To lowest order in λ_L/r it is

$$v_\theta = C e^{-r/\lambda_L} \tag{53}$$

where C is independent of r. To find C, we use the fact that except for the initial transient we can ignore the Maxwell term in Ampere–Maxwell's law Eq. (42), hence from Eq. (44)

$$v_\theta = -\frac{c}{4\pi n_s e}\frac{\partial H}{\partial r} \tag{54}$$

and replacing in Eq. (51) and using Eq. (52)

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}(rH) - \frac{1}{\lambda_L^2}(H - H_c) = 0 \tag{55}$$

so that $H - H_c$ and v_θ obey the same equation. To lowest order in λ_L/r again the solution is

$$H(r) = H_c - C' e^{-r/\lambda_L} \,. \tag{56}$$

Now we use the condition $H(r = r_0) = 0$ to get

$$C' = e^{r_0/\lambda_L} H_c \tag{57}$$

hence

$$H(r) = H_c(1 - e^{(r_0 - r)/\lambda_L}) \,, \tag{58}$$

the same as Eq. (18). Replacing Eq. (58) in Eq. (54) we finally obtain

$$v_\theta = -\frac{c}{4\pi n_s e \lambda_L} H_c e^{(r_0 - r)/\lambda_L} \,, \tag{59}$$

i.e. the same as Eq. (20), with $\lambda = \lambda_L$.

Using Eq. (52), Eq. (59) can also be written as

$$v_\theta = -\frac{e\lambda_L}{m^* c} H_c e^{(r_0 - r)/\lambda_L} \,. \tag{60}$$

Note that London's equation for superconductors is

$$\vec{\nabla} \times \vec{v} = -\frac{e}{m^* c}\vec{H} \,, \tag{61}$$

so in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) = -\frac{e}{m^* c} H = \frac{e}{m^* c} H_c (e^{(r_0 - r)/\lambda_L} - 1) \,, \tag{62}$$

while Eq. (60) is, to the lowest order in λ_L/r

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta) = \frac{e}{m^*c}H_c e^{(r_0-r)/\lambda_L}\,, \tag{63}$$

so the velocity field of our perfect conductor definitely does *not* satisfy London's equation.

8. Effective Mass Reduction

As discussed in Sec. 5, the outward motion of electrons corresponds to both effective mass and bare mass flowing out, while the outward motion of holes corresponds to effective mass flowing out and bare mass flowing in. So the process shown in Fig. 4 results in no bare mass flowing out, but there is a net outflow of effective mass.

For an electron in Bloch state $\vec{k}$ with band energy ϵ_k, we define the effective mass m_k^* by

$$\frac{1}{m_k^*} = \frac{1}{\hbar^2}\frac{\partial^2\epsilon_k}{\partial k^2} \tag{64}$$

assuming there is no angular dependence for simplicity. For a given band we can define an effective mass density by

$$\rho_{m^*} = \int_{occ}\frac{d^3k}{4\pi^3}m_k^*\,, \tag{65}$$

where the integral is over the occupied (by electrons) states in the band. We can also of course define a bare mass density

$$\rho_m = \int_{occ}\frac{d^3k}{4\pi^3}m_e\,. \tag{66}$$

Both ρ_m and ρ_{m^*} are zero for an empty band, for a full band $\rho_{m^*} = 0$ and $\rho_m \neq 0$. We can also define the associated mass and effective mass currents

$$\vec{j}_m = \int_{occ}\frac{d^3k}{4\pi^3}m_e\vec{v}_k\,, \tag{67}$$

$$\vec{j}_{m^*} = \int_{occ}\frac{d^3k}{4\pi^3}m_k^*\vec{v}_k \tag{68}$$

with

$$\vec{v}_k = \frac{1}{\hbar}\frac{\partial\epsilon_k}{\partial\vec{k}}\,. \tag{69}$$

Note that the effective mass current density can also be written in the simple form

$$\vec{j}_{m^*} = \int_{occ}\frac{d^3k}{4\pi^3}\left(\frac{\partial\epsilon_k}{\partial\vec{v}_k}\right). \tag{70}$$

Both real mass and effective mass currents satisfy continuity equations:

$$\vec{\nabla}\cdot\vec{\jmath}_m + \frac{\partial\rho_m}{\partial t} = 0\,. \tag{71a}$$

$$\vec{\nabla} \cdot \vec{\jmath}_{m^*} + \frac{\partial \rho_{m^*}}{\partial t} = 0 . \tag{71b}$$

When there is conduction in more than one band, the contributions from each band to the densities and currents simply add. For the case under consideration here, we have

$$\vec{\nabla} \cdot \vec{\jmath}_{m,t} = 0 , \tag{72a}$$

$$\vec{\nabla} \cdot \vec{\jmath}_{m^*,t} = -\frac{\partial \rho_{m^*,t}}{\partial t} \neq 0 , \tag{72b}$$

where by the subindex t (total) we mean the sum over both bands shown in Fig. 6.

We assume that the bands in Fig. 6 are respectively close to empty and close to full, so that the effective mass can be taken to be independent of k for the occupied states for the almost empty band and for the unoccupied states for the almost full band. Near the top of the band m_k^* [Eq. (64)] is negative and we define the effective mass of holes near the top of the band as

$$m_h^* = -m_k^* \tag{73}$$

and for electrons near the bottom of the band

$$m_e^* = +m_k^* . \tag{74}$$

We have then for both bands, denoted by e and h

$$\rho_{m^*}^e = \int_{occ} \frac{d^3k}{4\pi^3} m^* \equiv n_e m_e^* , \tag{75a}$$

$$\rho_{m^*}^h = \int_{unocc} \frac{d^3k}{4\pi^3} m^* \equiv n_h m_h^* \tag{75b}$$

and furthermore assume $n_e = n_h = n_s$, so that no net outflow of mass occurs, with n_s the superfluid density.

In the process shown in Fig. 4 there is a net outflow of n_e electrons and n_h holes, carrying out effective mass m_e^* and m_h^*, respectively, per carrier, in the process where the magnetic field is expelled, i.e. in the process where the system goes from the normal to the superconducting state. This implies

$$\Delta\rho_{m^*} \equiv \rho_{m^*}^n - \rho_{m^*}^s = n_s(m_e^* + m_h^*) \tag{76}$$

where the superscripts n and s refer to normal and superconducting states. Therefore,

$$\rho_{m^*}^s = \rho_{m^*}^n - n_s(m_e^* + m_h^*) . \tag{77}$$

which says that the effective mass per carrier is lowered by $(m_e^* + m_h^*)$ when the system goes from the normal to the superconducting state and expels the magnetic field by expelling electrons and holes.

Now recall that n_s, the superfluid density in the superconducting state, equals the density of charge carriers in the normal state, which is n_e for a band close to

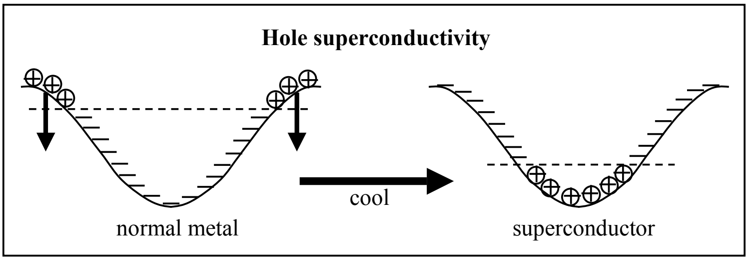

Fig. 8. In the normal state of the metal, the band is almost full, with n_h holes per unit volume that have effective mass m_h^*. As the metal becomes superconductor, the holes move from the top to the bottom of the band. This gives a reduction in the effective mass density of $n_s(m^*e + m_h^*)$, with m_e^* the effective mass of electron carriers at the bottom of the band.

empty and is n_h for a band close to full. Therefore, our result Eq. (77) is represented with what is shown in Fig. 8. If the normal metal has a band that is almost full, with n_h hole carriers with effective mass m_h^* (left panel), its effective mass density is

$$\rho^n_{m^*} = \int_{occ} \frac{d^3k}{4\pi^3} m_k^* = -\int_{unocc} \frac{d^3k}{4\pi^3} m_k^* = n_h m_h^* \,. \tag{78}$$

The right panel of Fig. 8 depicts n_h empty states at the bottom of the band. The effective mass density for that situation is

$$\rho^s_{m^*} = \int_{occ} \frac{d^3k}{4\pi^3} m_k^* = -\int_{unocc} \frac{d^3k}{4\pi^3} m_k^* = -n_h m_e^* \,. \tag{79}$$

Therefore, Eq. (77) is satisfied.

As seen in Fig. 8, the physics we are finding *requires* that in the normal state the charge carriers are holes, as proposed in the theory of hole superconductivity.[6] Furthermore, Fig. 8 indicates that when the system goes from normal metal to superconductor the holes near the top of the band "Bose condense" into states at the bottom of the band. We discuss this further in Sec. 11.

9. Angular Momentum Conservation

The important issue of angular momentum conservation needs to be addressed. In the process shown in Fig. 4, the final state has angular momentum given by[25]

$$L = (2\pi R\lambda_L h n_s) m_e v_s R = \frac{m_e c}{2e} R^2 h H_c \,. \tag{80}$$

How did electrons acquire this angular momentum,[a] and how is angular momentum conserved?

[a]The magnitude of the angular momentum given by Eq. (80), for example for a cylinder of radius $R = 1$cm, height $h = 5$ cm and magnetic field $H_c = 200$ G is $L = 2.84\ \text{mg} \cdot \text{mm}^2/s$.

For the discussion in Sec. 6 we assumed initial conditions so that only electrons have azimuthal velocity. However, let us consider first the simpler situation where the initial velocity is zero for both negative and positive charges.

As the perfectly conducting fluid starts moving outward, after a time $t_0 \sim \lambda_L/\dot{r}_0$ negative and positive charges near the inner boundary have acquired equal and opposite azimuthal velocities due to the action of the magnetic Lorentz force, giving rise to the azimuthal current density Eq. (21) as the sum of both contributions. The total angular momentum is thus zero. As the fluid moves out, both negative and positive charges increase their angular momentum, and at the end they both attain half the value Eq. (20), but their sum remains zero at all times. Thus conservation of angular momentum follows naturally in this scenario. However, it still needs to be explained how the charges increase their angular momentum as the fluid moves out, given that we said in Sec. 6 that the electric and magnetic forces in the azimuthal direction are balanced for both negative and positive charges (Fig. 7).

The reason is, the treatment given in Sec. 6 was approximate, valid to lowest order in λ_L/r. Recall also that we found for example that Eq. (9) was satisfied only to lowest order in λ_L/r. An exact treatment is more complicated and requires the use of Bessel functions. One finds that in fact the electric and magnetic forces are not exactly balanced, the electric force is slightly larger, providing the necessary torque so that the azimuthal velocity does not slow down but rather stays constant as the fluid moves out, thus imparting the increasing angular momentum to the currents.

Going back to the scenario where only electrons have azimuthal velocity shown in Fig. 7, it would require a very artificial initial condition: that both electrons and holes initially have azimuthal velocity in counterclockwise direction given by half the value Eq. (20), so that in the outward motion the Lorentz force causes the holes to stop and the electrons to double their initial velocity. This is not what we say happens in the Meissner effect.

In Sec. 10, we will discuss what really happens in the Meissner effect according to the theory of hole superconductivity.[6] But it should be clear from the discussion here and in Sec. 6 that the essential physics of magnetic field expulsion follows simply from these magnetohydrodynamic considerations.

10. What Really Happens

The process depicted in Fig. 4 shows the essential physics of what we argue is required to expel the magnetic field in the normal-superconductor transition. But it is only a caricature of what really happens, it cannot be the reality. In particular, holes flowing *out of* the outer boundary of the annulus in Fig. 4 implies that electrons are flowing *into* the outer boundary of the annulus. But where did those electrons come from?

The theory of hole superconductivity provides the answer. We review the physics here, discussed in earlier references.[25–32] It requires that the normal state charge

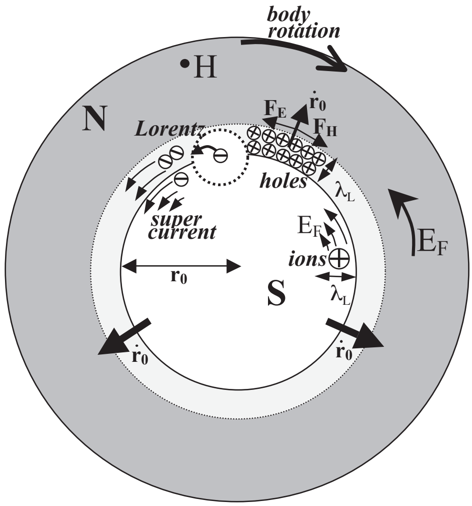

Fig. 9. Meissner effect according to the theory of hole superconductivity. As the normal electrons become superconductive, their orbits (dotted circle) expand, and the resulting Lorentz force propels the supercurrent. An outflow of hole carriers moving in the same direction as the phase boundary restores charge neutrality and transfers momentum to the body as a whole to make it rotate clockwise, without any scattering processes.

carriers are in a band that is almost full, with hole concentration n_s that will become the superfluid density.

First, the theory predicts that when electrons condense into the superconducting state their orbits expand from a microscopic radius to radius $2\lambda_L$.[26] The radius is determined by quantization of angular momentum.[27]

This orbit expansion is equivalent to an *outflow* of the electron negative charge a distance λ_L. To preserve charge neutrality, an *inflow* of normal electrons has to occur over that distance. These normal electrons are in a band that is almost full, so they represent an inflow of negative effective mass carriers, or equivalently an outflow of holes over the same distance. The process is depicted in Fig. 9.

The electric and magnetic Lorentz forces acting on the holes are balanced as shown in Fig. 9, just as we showed in Fig. 7 in our "caricature" process. The holes move out radially at speed $\dot{r}_0$, the speed of motion of the phase boundary, with no azimuthal velocity.

On the electrons, electric and magnetic forces are not balanced. We assume the orbit expansion occurs at great speed (much larger than $\dot{r}_0$). In expanding the orbit to radius $2\lambda_L$ the electrons acquire azimuthal (counterclockwise) velocity

$$v_s = -\frac{e\lambda_L}{m^* c} H_c \tag{81}$$

driven by the magnetic Lorentz force, with the electric Faraday force in the opposite direction having negligible effect.[25]

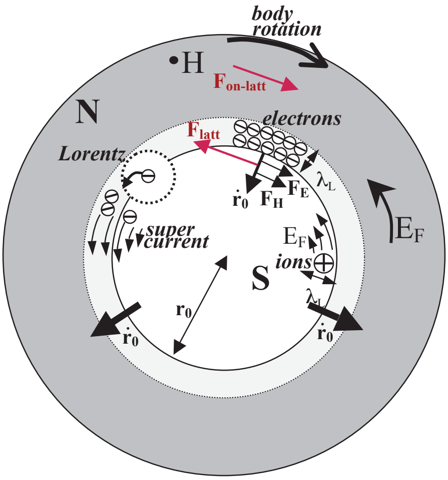

Fig. 10. Figure 9 redrawn replacing the outflowing holes by inflowing electrons. The electric and magnetic forces on inflowing electrons F_E and F_H point in the same direction. Since the motion is radial, this implies that another force must exist, $F_{\rm latt}$, exerted by the periodic potential of the ions on the charge carriers. By Newton's third law, an equal and opposite force is exerted by the charge carriers on the ions, $F_{\text{on-latt}}$, that makes the body rotate.

The Faraday electric field is slightly different than in our simple model of Sec. 6, it is given by

$$\vec{E} = \frac{\dot{r}_0}{c} H_c e^{(r-r_0)/\lambda_L} \hat{\theta} \tag{82a}$$

for $r \le r_0$, and

$$\vec{E} = \frac{\dot{r}_0}{c} \frac{r_0}{r} H_c \hat{\theta} \tag{82b}$$

for $r \ge r_0$. The azimuthal speed of electrons is

$$\vec{v}_s(r) = -\frac{e\lambda_L}{m^* c} H_c e^{(r-r_0)/\lambda_L} \hat{\theta} \tag{83}$$

for $r \le r_0$ and zero for $r > r_0$ (except for a small normal current induced by E[25]). Note that the speed increases with r, in contrast to the situation in Sec. 6 where it decreases (see Fig. 7). As the phase boundary moves further out, the Faraday electric field slows down the azimuthal velocity Eq. (83) as the given point r gets further away from the phase boundary, and both $\vec{E}$ and $\vec{v}_s$ go to zero in the deep interior.[25]

Figure 10 shows the same process as Fig. 9 with the outflowing holes replaced by inflowing electrons. It clarifies the important issue of angular momentum balance.[25,30] As the electrons in the expanding orbits acquire their azimuthal speed their increasing angular momentum has to be compensated by the body as a whole acquiring angular momentum in opposite direction. This happens through the backflow of electrons with negative effective mass, i.e. outflow of holes. The lattice exerts an azimuthal force $F_{\rm latt}$ on these electrons, and in turn these electrons exert a force on the lattice $F_{\rm on-latt}$ that transfers angular momentum to the body without any scattering processes that would lead to irreversibility. It is essential that the normal state charge carriers are holes. This is a key issue explained in detail in Refs. 25, 29 and 31.

In summary, "what really happens" is not exactly the same but very similar in spirit to the "caricature" process shown in Fig. 4 and discussed in Sec. 6, which could be understood simply using (almost) purely classical concepts. The difference here is that it is not the same electrons and holes that move continuously out, as in Fig. 4. Rather, electrons right outside the phase boundary move out a distance λ_L when they enter the superconducting state, and normal electrons from a distance up to λ_L outside the phase boundary move in. The region inside the phase boundary ends up in the superconducting state, having expelled $n_s m_e^*$ and absorbed $-n_s m_h^*$ effective mass density in the process, or equivalently having lowered its effective mass density by $(m_e^* + m_h^*)$ per normal state carrier, as we discussed in Sec. 8.

11. The Physics of Hole Superconductivity

We have described the motion of magnetic field lines when a normal metal turns superconducting using concepts used in describing the magnetohydrodynamics of conducting fluids, and in particular Alfven's theorem. Let us recapitulate our reasoning.

Starting from the observation that perfectly conducting fluids drag magnetic field lines with them when they move, we suggested that the moving field lines in the Meissner effect are dragged by a perfectly conducting fluid. We argued that this fluid has to be both charge-neutral and mass-neutral in order to not generate charge nor mass imbalance. We concluded that in order for this to happen it is necessary that the system expels the same concentration of electrons and holes.

We found that this implies that when the system goes from normal to superconducting and expels a magnetic field it also expels effective mass, so the effective mass in the system is reduced in going from the normal to the superconducting state. The amount of effective mass reduction per superfluid carrier was found to be independent of the magnitude of the magnetic field expelled. This then leads us to the general conclusion that when a system turns superconducting the carriers lower their effective mass, whether or not a magnetic field is present.

It is interesting to note that back in 1950 John Bardeen proposed a model of superconductivity which had as an essential ingredient a reduction of the carriers' effective mass upon entering the superconducting state.[33] However the model did not include the pairing concept, and in the subsequent BCS theory the effective mass reduction concept was not incorporated.

Within the theory of hole superconductivity[6] the interaction that gives rise to pairing is a correlated hopping term Δt in the effective Hamiltonian that increases the mobility of carriers when they pair,[34,35] or in other words decreases their effective mass. Superconductivity is driven by lowering of kinetic energy or equivalently by effective mass reduction. There is a lowering of the effective mass of Cooper pairs relative to the effective mass of the normal carriers,[36–38] and this gives rise to a London penetration depth that is smaller than expected from the normal state effective mass.[39] This in turn leads to an apparent violation[40,41] of the low frequency optical conductivity sum rule (Ferrell–Glover–Tinkham sum rule)[42,43] that was detected experimentally in several high T_c superconductors years after first predicted.[44–47]

More fundamentally the theory predicts that carriers "undress" in the transition from the normal to the superconducting state,[41,48,49] both lowering their effective mass and increasing their quasiparticle weight.[50] In a many-body system, the quasiparticle weight is inversely proportional to the effective mass, a highly dressed particle has both large effective mass and small quasiparticle weight and vice versa.[51] Clear experimental evidence for increase in the quasiparticle weight upon onset of superconductivity has been found in cuprate superconductors.[52]

Even more fundamentally, the theory predicts that carriers undress from *both* the electron–electron interaction and the electron–ion interaction.[53–56] In the normal state of the system when the band is almost full, i.e. when the normal state carriers are holes, carriers are "dressed" by the electron–ion interaction causing the electrons at the Fermi energy to have negative rather than positive effective mass. When the system turns superconducting, experiments and theory clearly show that the n_s superfluid carriers are "undressed" from the electron–ion interaction because they behave as electrons with negative charge.[58–63] For example, a rotating superconductor shows always a magnetic field in direction parallel, never antiparallel, to its angular velocity.[58]

The latter was understood to reflect the fact that the wavelength of carriers expands when they go from normal to superconducting.[56] Normal state carriers at the top of the band interact strongly with the discrete ionic potential and when they become superconducting and their wavelength expands they no longer "see" the discrete ionic potential, hence have "undressed" from it and behave as electrons rather than holes. More specifically, the wavelength expansion was found to result from electronic orbits expanding from a microscopic radius to radius $2\lambda_L$ in the transition. All of this led us to conclude that "holes turn into electrons" in the normal to superconducting transition.[65,66] Based on this physics, supported by quantitative calculations, Fig. 8 was proposed in 2010, Fig. 6 of Ref. 67. What this means for the wavefunction of the carriers is what is shown in Fig. 11. In the normal state, carriers at the Fermi energy are in "antibonding states", with highly oscillating wavefunction and high kinetic energy, while in the superconducting state they adopt the same wavefunction that electrons have near the bottom of the band in the normal state, i.e. bonding states, with smooth wavefunction and low kinetic energy. Figure 8 expresses this fact.

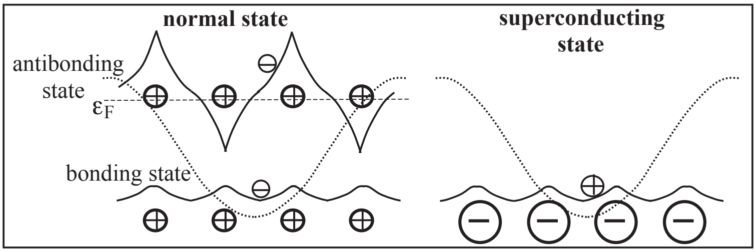

Fig. 11. When a band is nearly full, carriers at the Fermi energy (indicated by ϵ_F) are in antibonding states (left panel), with highly oscillating wavefunction and high kinetic energy. Carriers near the bottom of the band are in bonding states, with smooth wavefunction and low kinetic energy. According to the finding in Fig. 8, when a system becomes superconducting and expels electrons and holes, the wavefunction for the superconducting carriers becomes as shown in the right panel of the figure, a bonding state. The large circles with negative charge on the right panel represent the negatively charged ion with the orbital doubly occupied.

In this paper we have independently "rediscovered" Figs. 8 and 11 by finding that the Meissner effect requires normal state carriers of density n_s to lower their effective mass by $(m_e^* + m_h^*)$ as they become superconductive, or equivalently that they change their effective mass from $-m_h^*$ to m_e^*. This requires that the initial state has a band that is almost full, with hole carriers of mass m_h^*, and that in becoming superconductive, the holes move from the top to the bottom of the band as shown in Fig. 8.

The *requirement* that normal state carriers in a metal that can become a superconductor are holes rather than electrons[31] follows directly from this physics. There would be no way for carriers to lower their effective mass by $(m_e^* + m_h^*)$ starting with a normal state with electron carriers of effective mass m_e^*.

12. Dissipation

In the process shown in Fig. 4, magnetic field lines are carried out by a perfectly conducting fluid, and no dissipation is associated with that motion.[1] However, Joule heat is still generated due to the motion of magnetic field lines outside the region occupied by the perfectly conducting fluid. Let us calculate that. At a given time, the perfectly conducting fluid in Fig. 4 occupies an annulus $r_0 < r < r_1$, with

$$r_1^2 = r_0^2 + 2R\lambda_L\,. \tag{84}$$

The electric field for $r > r_1$ is

$$E(r) = \frac{r_0}{r}\frac{\dot{r}_0}{c}H_c \tag{85}$$

and the Joule heat dissipated per unit volume per unit time in the region $r_1 < r < R$ is

$$\frac{\partial w}{\partial t} = \sigma\frac{\dot{r}_0^2}{c^2}\frac{r_0^2}{r^2}H_c^2 \tag{86}$$

with σ the conductivity in the normal region. The total heat per unit time dissipated in the region $r_1 < r < R$ is

$$\frac{\partial W}{\partial t} = \int_{r_1}^{R} d^3r \frac{\partial w}{\partial t} = 2\pi h H_c^2 \frac{\sigma}{c^2} \dot{r}_0^2 r_0^2 \int_{r_1}^{R} dr \frac{1}{r} \tag{87}$$

and integrated over time

$$W = 2\pi h H_c^2 \frac{\sigma}{c^2} \dot{r}_0 \int_0^{R-\lambda_L} dr_0 r_0^2 \ln(R/r_1) \tag{88}$$

assuming for simplicity that $\dot{r}_0$ is time-independent.

Instead, let us assume that the magnetic field gets expelled through some unknown quantum mechanism that does not involve the motion of a perfectly conducting fluid, as in BCS. The same Eq. (87) applies with r_0 replacing r_1, hence

$$\frac{\partial W_0}{\partial t} = 2\pi h H_c^2 \frac{\sigma}{c^2} \dot{r}_0^2 r_0^2 \int_{r_0}^{R} dr \frac{1}{r} \tag{89}$$

and the total Joule heat dissipated is

$$W_0 = 2\pi h H_c^2 \frac{\sigma}{c^2} \dot{r}_0 \int_0^{R-\lambda_L} dr_0 r_0^2 \ln(R/r_0)\,. \tag{90}$$

Carrying out these integrals we find, to the lowest order in λ_L/R

$$W_0 = \frac{2\pi}{9} h R^3 H_c^2 \frac{\sigma}{c^2} \dot{r}_0 \tag{91}$$

and

$$W = W_0 - \Delta W\,, \tag{92}$$

with

$$\Delta W = 12 \frac{\lambda_L}{R} W\,, \tag{93}$$

so the Joule heat dissipated is less when the process occurs through motion of a perfectly conducting fluid as in Fig. 4. Alternatively, for the same amount of Joule heat the process will be faster through the motion of a perfectly conducting fluid.

Something similar occurs for the Meissner effect. If we assume that the expulsion of magnetic field occurs without any radial motion of charge, as in BCS, the Joule heat dissipated is given approximately by[30]

$$Q_J^0 = \frac{H_c^2}{8\pi} \frac{2\pi\sigma}{c^2} R \dot{r}_0\,. \tag{94}$$

For a quantitative estimate, we assume σ is given by the Drude form $\sigma = ne^2\tau/m_e$ with τ the collision time, so $\sigma = c^2/(4\lambda_L^2)\tau$, $\dot{r}_0 = R/t_0$ with t_0 the time to expel the magnetic field, and with $R = 1$ cm, $\lambda_L = 500$ Å, Eq. (94) yields

$$Q_J^0 = \frac{H_c^2}{8\pi} (2 \times 10^{10}) \frac{\tau}{t_0} \tag{95}$$

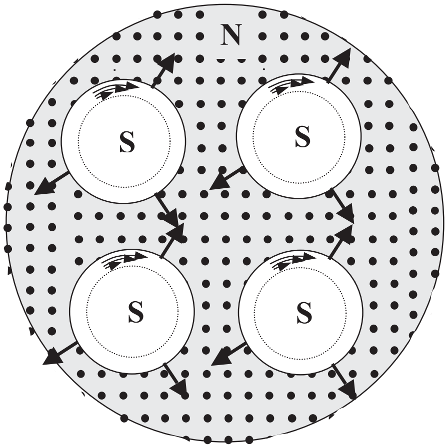

Fig. 12. Expulsion of magnetic field through nucleation of several superconducting domains that expand and merge. In the annuli of thickness λ_L adjacent to the phase boundaries supercurrent flows, a Faraday field exists, and Joule heat is dissipated according to the conventional theory.[57]

so for example for $\tau = 10^{-11}$ s at low temperatures in a pure crystal and $t_0 = 1$ s, Q_J^0 is 20% of the superconducting condensation energy $H_c^2/8\pi$. In addition, within the conventional theory there will be Joule heat originating in the superconducting region of thickness λ_L next to the phase boundary, due to the action of the Faraday field on the normal electrons in that region,[57] given by

$$\Delta Q_J = 4\frac{\sigma'}{\sigma}\frac{\lambda_L}{R}Q_J^0 \tag{96}$$

with σ' the conductivity of normal electrons (of density n_n) in the superconducting region, $\sigma'/\sigma \sim n_n/(n_n + n_s)$. The correction Eq. (96) is small if $\lambda_L \ll R$. However note that we are assuming that the flux expulsion occurs through the expansion of a single domain as shown in Fig. 9. Consider instead the more realistic scenario where the superconducting phase nucleates at several different points simultaneously, creating several domains that expand expelling magnetic field, as shown in Fig. 12. Now we need to add the regions of thickness λ_L within each domain where dissipation will take place according to the conventional theory for the calculation of ΔQ_J. This corresponds roughly to replacing R by R/N in Eq. (96), with N the number of domains. So we conclude that if the transition occurs through nucleation of many domains the Joule heat dissipated as the magnetic flux is expelled will be drastically increased compared to our scenario where flux expulsion occurs with accompanying fluid motion. Alternatively, for a given Joule heat the total time for the field to be expelled will be much smaller when fluid motion and many domains are involved.

It should be possible to observe this experimentally. Experimentally it should be possible to realize the different domain scenarios by setting up appropriate temperature or magnetic field gradients.

13. Discussion

None of the physics discussed in this paper is part of the conventional theory of superconductivity.[5] About the Meissner effect the conventional theory simply says that magnetic field lines move out because the superconducting state with magnetic field excluded has lower energy than the normal state with magnetic field inside. The dynamics of the Meissner effect and of the related effect of magnetic field generation when a rotating normal metal becomes superconducting (London field)[58,68] have not been explained within the conventional theory. The BCS "proof" of the Meissner effect[5] discussed in Sec. 2 starts with the system in the superconducting state and applies a magnetic field as a small perturbation, which is *not* the physics of the Meissner effect.[28]

It is important to remember that the laws of classical physics that we used in this paper always act, whether or not "quantum mechanics" also plays a role. Specifically, in addition to explaining how 'quantum mechanics' causes magnetic field lines to be expelled, the conventional theory has to explain how angular momentum is conserved and how the process overcomes the laws of classical physics that say that magnetic field lines have great difficulty in moving through conducting fluids, the more so the more conducting the fluid is, and that energy is dissipated in the process, and entropy is generated. The normal-superconductor transition in a magnetic field is a reversible phase transformation that occurs without entropy generation in an ideal situation. Entropy is not generated when magnetic field lines move following the motion of a perfectly conducting fluid, while entropy is generated when magnetic field lines cut across a conducting fluid, whether or not quantum mechanics plays a role. Within our theory, entropy is not generated locally around the phase boundary when the phase boundary is displaced, while it would be within the conventional theory. Based on this we have proposed that Alfven waves[2] should propagate along normal-superconductor phase boundaries if our theory is valid[69] and not if the conventional theory is valid.

Within the conventional theory the only thing that flows out when a system goes from normal to superconducting and expels a magnetic field is "phase coherence". Nobody has explained even qualitatively how this abstract concept explains the physical processes that take place, that at face value appear to violate fundamental laws of physics, namely the law of inertia, Faraday's law, conservation of angular momentum and conservation of entropy in reversible processes.[25,28,30,32]

Instead, in this paper, we have argued that magnetohydrodynamics strongly suggests that the Meissner effect in superconductors is associated with outflow of a perfectly conducting fluid in the normal-superconductor transition; that this perfectly conducting fluid needs to be composed of electrons and holes, to preserve charge neutrality and mass homogeneity; that electrons becoming superconducting flow out, and there is a backflow of normal antibonding electrons equivalent to an outflow of normal holes, and that momentum is conserved by holes transferring it to the body as a whole. The process as we describe is reversible, as required by thermodynamics, and satisfies the fundamental laws of physics. In this paper we described the process in more detail than in earlier work[25,28] and unexpectedly found that it leads to a lowering of effective mass in going from normal to superconducting, in unexpected agreement with what the theory of hole superconductivity has predicted for the last 30 years and was found experimentally in some high-temperature superconductors.[44–47,52] We also found in this paper that the process requires normal state carriers to be hole-like for yet another reason that adds to the many other reasons found in earlier work,[6] and in contrast to the conventional theory of superconductivity that is electron-hole symmetric.[5] Macroscopic phase coherence also follows naturally from this physics.[56,66]

The "population inversion" that we found in Fig. 8 is reminiscent of what occurs in laser physics. It is interesting that in both realms it is associated with the establishment of macroscopic quantum phase coherence.

In Ref. 67, we presented many other reasons for why the scenario shown in Figs. 8 and 11 captures the essence of superconductivity. We argue that the fact that in this paper we have "rediscovered" Figs. 8 and 11 from an entirely different argument strongly supports the validity of this theoretical framework to describe the real world. None of the commonly used model Hamiltonians[70–73] to describe correlated electrons in solids capture this physics. It represents a radical departure from the conventional theory of superconductivity, where it is assumed that carriers establish correlations between each other when they become superconducting but do not change their intrinsic character, i.e. their wavefunction. Instead, in our scenario carriers change their most essential characteristics, their quasiparticle weight and effective mass, because their wavefunction changes, through the complete redistribution of energy level occupation depicted in Fig. 8. Note that in the Mott-Hubbard transition as described by dynamical mean field theory[74] electrons also change their quasiparticle weight and effective mass in entering the low-temperature Mott state.[75] However, contrary to the physics discussed here, in that case the effective mass increases rather than decreases at low temperatures. Another situation where the effective mass decreases at low temperatures when the system undergoes a phase transition is proposed to be for metallic ferromagnetism.[38]

As shown in Ref. 67 and earlier papers, this physics also leads to a slight charge inhomogeneity in the ground state of superconductors,[76] with more negative charge near the surface and more positive charge in the interior, and to macroscopic zero point motion in the form of a spin current flowing near the surface of superconducting bodies in the absence of applied fields.[54,77,78]

We suggest that a valid microscopic theory of superconductivity has to be consistent with these findings. The currently accepted conventional theory of superconductivity[5] certainly is not.[79,80]

Appendix A. Alfven's Theorem

We prove Alfven's theorem for the case of interest in this paper shown in Fig. 4. Figure 13 shows a slice of our cylinder perpendicular to its axis evolving in time. The full lines show the outer and inner perimeters of the perfectly conducting fluid at time t, denoted by P_1, P_2, which are boundaries to the surfaces S_1 and S_2 which become S'_1 and S'_2 at time $t + dt$, bounded by the dotted lines. We want to show that the magnetic flux through S_1 and S'_1 are the same, as well as that through S_2 and S'_2. This then implies that the flux through the annulus does not change, and also that the flux through the central region inside the annulus remains zero.

The change in magnetic flux through one of these surfaces, $d\phi_i$ ($i = 1$ or 2) is

$$d\phi_i = \int_{S'_i} \vec{B}(\vec{r}, t + dt) \cdot d\vec{a} - \int_{S_i} \vec{B}(\vec{r}, t) \cdot d\vec{a} \tag{A1}$$

Using

$$\vec{B}(\vec{r}, t + dt) = \vec{B}(\vec{r}, t) + \frac{\partial B(\vec{r}, t)}{\partial t} dt \tag{A2}$$

we have

$$d\phi_i = \int_{dS_i} \vec{B}(\vec{r}, t) \cdot d\vec{a} + dt \int_{S_i} \frac{\partial B(\vec{r}, t)}{\partial t} \cdot d\vec{a} \tag{A3}$$

with $dS_i = S'_i - S_i$. The differential of area is

$$d\vec{a} = \vec{v} \times d\vec{\ell} \tag{A4}$$

where $\vec{v}$ is the velocity of the fluid. Using Eq. (A4) for the first integral and Eq. (9) for the second integral in Eq. (A3) we obtain

$$d\phi_i = dt \int_{P_i} \vec{B}(\vec{r}, t) \cdot (\vec{v} \times d\vec{\ell}) + dt \int_{S_i} \vec{\nabla} \times (\vec{v} \times \vec{B}) \cdot d\vec{a} \tag{A5}$$

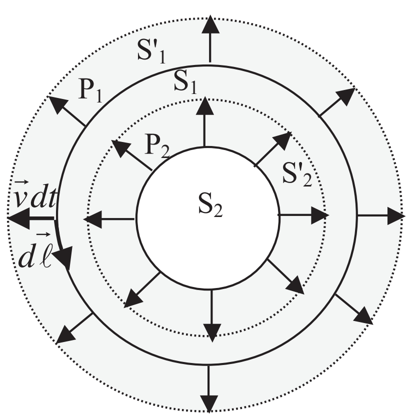

Fig. 13. Outward motion of perfectly conducting region. P_1 and P_2 are the outer and inner boundaries of the perfectly conducting region, indicated by full circles. dS_1 (dS_2) is the area bounded by the outer (inner) full circle and the outer (inner) dotted circle.

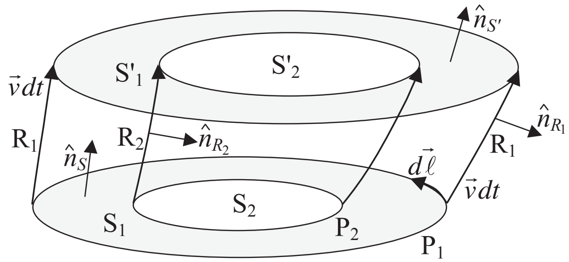

Fig. 14. Alfven's theorem for a more general geometry than in Fig. 12. Surfaces S_1 and S_2 bounded by perimeters P_1 and P_2 evolve to S_1' and S_2' in time dt following the motion of the fluid. R_1 and R_2 denote the ribbons connecting the surfaces under time evolution. The magnetic flux through the grey region stays the same for all times.

and permuting factors in the first term and using Stokes' theorem in the second term

$$d\phi_i = dt \int_{P_i} (\vec{B} \times \vec{v}) \cdot d\vec{\ell} + dt \int_{P_i} (\vec{v} \times \vec{B}) \cdot d\vec{\ell} = 0\,. \tag{A6}$$

This proves that magnetic field lines do not cross neither the outer nor the inner boundary as the annulus of perfectly conducting fluid moves outward. Magnetic field lines are frozen into the annulus and move out with it, pushing out magnetic field lines outside and leaving the interior field free.

For a more general geometry where the fluid velocity is not parallel to the area being considered, as shown in Fig. 14 the proof is only slightly more complicated. In addition to the flux through the surfaces S_1 and S_2 we need to consider also the flux through the ribbons R_1 and R_2 shown in Fig. 14. It can be shown that here also the flux through the multiply connected surface bounded by P_1 and P_2 is invariant under time evolution, and the flux in the interior of P_2 remains zero at all times.

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References

1. P. A. Davidson, *An Introduction to Magnetohydrodynamics* (Cambridge University Press, Cambridge, 2001).
2. H. Alfven, *Nature* **150** (1942) 405; *Arkiv foer Matematik, Astronomi och Fysik* **39** (1943) 2.
3. P. H. Roberts, Alfven's Theorem and the Frozen Flux Approximation, in D. Gubbins and E. Herrero-Bervera (eds.), *Encyclopedia of Geomagnetism and Paleomagnetism* (Springer, Dordrecht, 2007).
4. M. G. Kivelson *et al.*, *Science* **289** (2000) 1340.
5. M. Tinkham, *Introduction to Superconductivity* (McGraw Hill, New York, 1996).

6. References in https://jorge.physics.ucsd.edu/hole.html.
7. J. Bardeen, L. N. Cooper and J. R. Schrieffer, *Phys. Rev.* **108** (1957) 1175.
8. P. W. Anderson, *Phys. Rev.* **110** (1958) 827.
9. G. Rickayzen, *Phys. Rev.* **111** (1958) 817; *Phys. Rev. Lett.* **2** (1959) 90; *Phys. Rev.* **115** (1959) 795.
10. G. Wentzel, *Phys. Rev.* **111** (1958) 1488; *Phys. Rev. Lett.* **2** (1959) 33.
11. D. Pines and J. R. Schrieffer, Phys. Rev. Lett. **1** (1958) 407.
12. R. M. May and M. R. Schafroth, *Phys. Rev.* **115** (1959) 1446.
13. Y. Nambu, *Phys. Rev.* **117** (1960) 648.
14. L. P. Kadanoff and P. C. Martin, *Phys. Rev.* **124** (1961) 670.
15. D. A. Uhlenbrock and B. Zumino, *Phys. Rev.* **133** (1964) A350.
16. H. Frahm, S. Ullah and A. T. Dorsey, *Phys. Rev. Lett.* **66** (1991) 3067.
17. A. T. Dorsey, *Ann. Phys.* **233** (1994) 248.
18. F. Liu, M. Mondello and N. Goldenfeld, *Phys. Rev. Lett.* **66** (1991) 3071.
19. A. Schmid, *Phys. Kondens. Mater.* **5** (1966) 302.
20. E. Abrahams and T. Tsuneto, *Phys. Rev.* **152** (1966) 416.
21. M. Cyrot, *Rep. Prog. Phys.* **36** (1973) 103.
22. N. W. Ashcroft and N. D. Mermin, *Solid State Physics*, Chap.12 (Saunders College Publishing, Fort Worth, 1976).
23. T. E. Faber, *Nature* **164** (1949) 277.
24. M. von Laue, *Theory of Superconductivity* (Academic Press, New York, 1952).
25. J. E. Hirsch, *Phys. Rev. B* **95** (2017) 014503.
26. J. E. Hirsch, *Europhys. Lett.* **81** (2008) 67003.
27. J. E. Hirsch, *Europhys. Lett.* **113** (2016) 37001.
28. J. E. Hirsch, *Phys. Scripta* **91** (2016) 035801.
29. J. E. Hirsch, *Europhys. Lett.* **114** (2016) 57001.
30. J. E. Hirsch, *Ann. Phys.* **373** (2016) 230.
31. J. E. Hirsch, Why only hole conductors can be superconductors, in *Proc. SPIE 10105, Oxide-based Materials and Devices VIII, 101051V* (2017).
32. J. E. Hirsch, *Int. J. Mod. Phys. B* **32** (2018) 1850158.
33. J. Bardeen, *Phys. Rev.* **59** (1941) 928; *Phys. Rev.* **79** (1950) 167; *Phys. Rev.* **80** (1950) 567; *Phys. Rev.* **81** (1951) 823.
34. J. E. Hirsch and F. Marsiglio, *Phys. Rev. B* **39** (1989) 11515.
35. J. E. Hirsch, *Physica C* **158** (1989) 326.
36. J. E. Hirsch and F. Marsiglio, *Physica C* **162–164** (1989) 1451.
37. J. E. Hirsch, *Physica C* **179** (1991) 317.
38. J. E. Hirsch, *Physica C* **341–348** (2000) 211.
39. J. E. Hirsch and F. Marsiglio, *Phys. Rev. B* **45** (1992) 4807.
40. J. E. Hirsch, *Physica C* **199** (1992) 305.
41. J. E. Hirsch, *Physica C* **201** (1992) 347.
42. J. E. Hirsch and F. Marsiglio, *Physica C* **331** (2000) 150.
43. J. E. Hirsch and F. Marsiglio, *Phys. Rev. B* **62** (2000) 15131.
44. H. J. A. Molegraaf, C. Presura, D. van der Marel, P. H. Kes and M. Li, *Science* **295** (2002) 2239.
45. A. F. Santander-Syro, R. P. S. M. Lobo, N. Bontemps, Z. Konstantinovic, Z. Z. Li and H. Raffy, *Europhys. Lett.* **62** (2003) 568.
46. M. V. Klein and G. Blumberg, *Science* **283** (1999) 42; D. N. Basov *et al.*, *Science* **283** (1999) 49.
47. A. Charnukha, P. Popovich, Y. Matiks, D. L. Sun, C. T. Lin, A. N. Yaresko, B. Keimer and A. V. Boris, *Nature Comm.* **2** (2011) 219.

48. J. E. Hirsch, *Phys. Rev.B* **62** (2000) 14487; *Phys. Rev. B* **62** (2000) 14498.
49. J. E. Hirsch, *Phys. Rev. B* **66** (2002) 064507.
50. J. E. Hirsch, Quasiparticle Undressing: A New Route to Collective Effects in Solids, in *Concepts in Electron Correlation*, eds. A. C. Hewson and V. Zlatic, NATO Science Series (Series II: Mathematics, Physics and Chemistry), Vol. 110, (Springer, Dordrecht, 2003), pp. 371–380.
51. R. D. Mattuck, *A Guide to Feynman Diagrams in the Many-Body Problem* (McGraw-Hill, New York, 1976).
52. H. Ding *et al.*, *Phys. Rev. Lett.* **87** (2001) 227001.
53. J. E. Hirsch, *Phys. Rev. B* **68** (2003) 012510.
54. J. E. Hirsch, *Phys. Lett. A* **309** (2003) 457.
55. J. E. Hirsch, *AIP Conf. Proc.* **695** (2003) 21.
56. J. E. Hirsch, *Phys. Rev. B* **71** (2005) 104522.
57. J. E. Hirsch, arXiv:2001.07509 (2020).
58. A. F. Hildebrandt, *Phys. Rev. Lett.* **12** (1964) 190.
59. R. H. Pry, A. L. Lathrop and W. V. Houston, *Phys. Rev.* **86** (1952) 905.
60. J. Bok and J. Klein, *Phys.Rev.Lett.* **20** (1968) 660.
61. T. D. Morris and J. B. Brown, *Physica* **55** (1971) 760.
62. Y. N. Chiang and O. G. Shevchenko, *Low Temp. Phys.* **22** (1966) 513.
63. C. J. Adkins and J. R. Waldram, *Phys. Rev. Lett.* **21** (1968) 76.
64. J. E. Hirsch, *J. Phys. Cond. Matt.* **20** (2008) 235233.
65. J. E. Hirsch, *Int. J. Mod. Phys. B* **17** (2003) 3236.
66. J. E. Hirsch, *Int. J. Mod. Phys. B* **25** (2011) 1173.
67. J. E. Hirsch, *Int. J. Mod. Phys. B* **23** (2009) 3035.
68. J. E. Hirsch, *Ann. Phys.* **531**(10) (2019) 1900212, doi:10.1002/andp.201900212.
69. J. E. Hirsch, *Physica C* **564** (2019) 42.
70. G. Kotliar and S. Y. Savrasov, Model Hamiltonians and first principles electronic structure calculations, in A. M. Tsvelik (ed.), *New Theoretical Approaches to Strongly Correlated Systems*, NATO Science Series (Series II: Mathematics, Physics and Chemistry), Vol. 23 (Springer, Dordrecht, 2001, pp. 259–301.
71. R. Requist and E. K. U. Gross, *Phys. Rev. B* **99** (2019) 125114.
72. A. Klein and D. Jaksch, *Phys. Rev. A* **73** (2006) 053613 and references therein.
73. M. Daghofer, A. Moreo, J. A. Riera, E. Arrigoni, D. J. Scalapino and E. Dagotto, *Phys. Rev. Lett.* **101** (2008) 237004.
74. A. Georges, G. Kotliar, W. Krauth and M. J. Rozenberg, *Rev. Mod. Phys.* **68** (1996) 13.
75. X. Y. Zhang, M. J. Rozenberg and G. Kotliar, *Phys. Rev. Lett.* **70** (1993) 1666.
76. J. E. Hirsch, *Phys.Rev. B* **68** (2003) 184502.
77. J. E. Hirsch, *Phys. Rev. B* **71** (2005) 184521.
78. J. E. Hirsch, *Ann. Phys.* (*Berlin*) **17** (2008) 380.
79. J. E. Hirsch, *Phys. Scripta* **80** (2009) 035702.
80. J. E. Hirsch, La superconductividad bien entendida empieza con H, *Editorial Reverté*, Barcelona (2019), pp. 1–264.